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J. W. L. GLAISHER, M.A., F.R.S., President, in the Chair.

On the Determination of the Radius Vector in the Absolute Orbit of the Planets. By Prof. Hugo Gylden, Director of the Observatory, Stockholm.

The method of variation of constants, given by Lagrange, forms the theoretical foundation of astronomy at the present time. In cases where the mathematical problem cannot be rigorously solved we have, by its help, not only been enabled to compute the movements of the heavenly bodies by successive approximations, but have also succeeded in stating the nature of the motions in an intelligible manner. We seem also to have attained to something more precise than a mere rule for the computation of the effect of mutual attraction within definite intervals of time. The way in which this method has been hitherto applied in investigations on the theory of the movements of the heavenly bodies was not, however, in all respects the most appropriate for giving that general view of the laws of movement which should not be absent from a comprehensive scientific solution of the problem in question. The slight amount of success which results from the application of this method is explained by the fact that, if the approximations are carried out in a straightforward manner, they lead to developments in series according to powers of the time, and these series do not converge for any arbitrary magnitude of the variable; so that it is difficult to judge of the convergence of the approximations even when the development in powers of the time has been avoided by certain artifices.

It is well known that the method of variation of constants allows us a certain discretion; for a number of the equations from which the variations of the elements are to be computed can be fixed on at pleasure, and an arbitrary number of solutions can be found for the differential equations which, from a formal point of view, are distinct from one another, and which together contain a sufficient number of arbitrary constants to give us the place and velocity of a heavenly body at a given moment of time. In the application of this method to the planetary theory it has, until recently, been attempted to satisfy this condition as to the place and velocity of the planet by giving to the co-ordinates and to their first differentials with respect to the time, the same form in the later approximations as in the first.

The values, which in the first approximation appear as constants of integration, become in the later stage functions of the time, which must be evaluated step by step. Under these circumstances the orbits of the planets are regarded as Keplerian ellipses, whose elements, however, vary from instant to instant. But the variations of the elements cannot be regarded from so simple a point of view; little more can be said in description of them than that they are small in small intervals of time, but that if the time be taken sufficiently great they may become so considerable that the original ellipse is quite unrecognisable. The motion, the nature of which is thus intelligibly described, holds good strictly for a very short time; it can, however, be retained without actual practical inconvenience during several decades of years—perhaps for several centuries. However, the representation of the motion of the heavenly bodies, indicated above, is not sufficient from a scientific point of view, where it is desired to regard the phenomenon as independent of shortness or length of time; for the condition, that the co-ordinates and their differentials shall have the same form as in Kepler's theory of planetary motion, does not repose on a circumstance which should *à priori* give it the preference over all other methods. If actual analytical difficulties did not stand in the way, that condition ought to be replaced by another which would be appropriate to represent the nature of the planetary movements more completely than has hitherto been the case. A result of this would certainly be that a number of conceptions appropriate to Keplerian astronomy would be replaced by others. As may be however conjectured without analysis, the practical rules for computing the places of the planets would undergo very little alteration.

For many years then I have been convinced of the necessity of replacing the present point of view of the planetary theory by another which shall correspond in its actual features with the phenomenon of planetary motion during any arbitrary period. The conception of an absolute orbit and of absolute elements has now been established in consequence of the labour which I have devoted to this subject. The absolute orbit gives the path

of the actual motion very closely, so that the departure from exactness of the place computed in this orbit always remains of the order of the disturbing forces, and never assumes the character of a secular variation. In contrast with the elements of the osculatory or mean ellipse, which undergoes considerable variation under the influence of the secular disturbances, the elements of the absolute orbit are absolute constants, which are not subject to secular variations, otherwise than that the longitude is changed by the motion of the planet, of the apses, and of the nodes of its orbit.

Hence, the longitude must be given for a definite epoch. The three modular elements on which the mean distance, the excentricity, and the inclination depend, are, on the other hand, absolutely invariable, and serve for any moment of time whatever. For the representation of the co-ordinates in the absolute orbit I make use of certain functions which depend on the disturbing masses, but which do not vanish with them, but become constants corresponding to elliptic elements. For this reason I call these *the elementary functions*, and the individual terms, out of which these are formed, are the *elementary terms*.

The elementary terms have a form which is easily to be recognised; they are periodic, and their periods are either very long compared with the periodic time, form (A), or differ very little from that period, form (B). In the course of the developments, however, terms make their appearance which have the same form as the elementary terms, but differ from them in that they vanish with the disturbing masses. Such terms I call *subelementary terms*, and I say that they are of the first order when they involve, as a factor, the first power of a disturbing mass, and of the second order when they involve the square of a disturbing mass, or the product of two masses, &c. With reference to the excentricity, or to the constants which determine its magnitude, the elementary terms are of various orders, and the same holds with respect to the inclination. I reckon as of the first class an elementary term which contains the first power of a modulus of the excentricity; and of the second class terms which contain the squares or products in two dimensions of these moduluses, and so on.

It may be here pointed out that the terms of the form (A), whose periods are very long, belong to an even class, and the terms of the form (B) to an uneven class. If the succession of approximations is not appropriately arranged, terms may make their appearance in which the disturbing masses appear as divisors. Such terms must, of course, vanish from the result if the approximations are in general to converge and to lead to a real result. The appearance of terms of this kind always shows that the method in application, or the fundamental system of differential equations, was not the most appropriate; for the operations which lead to the vanishing of these terms are usually very troublesome, and the numerical results are always inexact,

because they arise from the differences of larger quantities. Terms which involve the disturbing masses as divisors I call *hyperclementary* terms. It would have been desirable that this name should not be required; but as such terms may appear, and as it would be easy to give equations of perturbation which necessarily lead to this complication of the problem, it is necessary to arm each method against the appearance of such terms. If no care is taken that the hyperclementary terms should not appear, or should not be made to vanish easily, the method in use, and the differential equations on which it is based, are certainly inappropriate. Also, if it cannot be proved that the hyperclementary terms must disappear, the conclusions derived from the method are of course untenable.

I have already published in several places investigations on the theory of the absolute orbit, or sketches of such investigations; in the following paper I shall show the connection of this theory with that of the variation of constants. This will afford the best means of finding points of comparison with the former theory of the motion of the heavenly bodies, the basis of which is furnished by the laws of Kepler. In place, however, of a complete comparison it must suffice to submit only the equation of the radius vector to an examination from this point of view; for this equation is especially characteristic of the new method of considering the subject.

At the time t , let r be the true radius vector, and v the true longitude in the orbit.

In the complete development of the theory of the absolute orbit, I do not usually make r dependent on v , but on an argument v_0 , which differs from v by a magnitude of the order of the disturbing force. For the present purpose, however, this difference is altogether unimportant, and we shall, therefore, always write v instead of v_0 . In place of r I now introduce a new function ρ , defined by

$$r = \frac{p}{1 + \rho},$$

where, however, p must not be regarded as constant, but has the signification of an elementary function. It is easy to see that, in the elliptic theory, p and ρ have the following meanings, viz. :—

$$p = a(1 - e^2); \quad \rho = e \cos(v - \varpi),$$

where a is the mean distance, e the excentricity, and ϖ the longitude of perihelion.

I denote the radius vector in the absolute orbit by (r) , and put

$$(r) = \frac{p}{1 + (\rho)}$$

finally I write

$$r = \frac{(r)}{1 + \frac{(\rho)}{a} \xi}$$

So that

$$\frac{a}{r} - \frac{a}{(r)} = \xi; \quad \rho - (\rho) = \frac{p}{a} \xi.$$

For the determination of ρ a differential equation of the second order may be found, which may be written as follows:—

$$\frac{d^2 \rho}{dv^2} + (1 - \beta + X) \rho = R.$$

In this equation β denotes a constant of the order of the perturbing forces, X and R functions of the same order of magnitude, which, however, for the present are not fully known. They contain, in fact, besides other unknown magnitudes, the function ρ .

If, however, we confine ourselves in the first approximation to the determination of the elementary terms of the first class, we may regard the functions X and R as known. After the determination of ρ corresponding to these values of X and R , we obtain more exact values of X and R , and repeat therewith the determination of ρ . Thus these functions are obtained by successive approximations. We here remark that X may in a certain sense be considered as arbitrary, for only those terms need be retained in X which, on multiplication by ρ , produce subelementary and elementary terms in the subsequent approximation.

We may further remark that, if all terms which contain uneven powers of ρ as factors are regarded as being collected in $X\rho$, both X and R (of course only in so far as they contain subelementary terms, or give rise to such) may be regarded as even functions of ρ .

In the above differential equation let us now substitute for ρ the value

$$(\rho) + \frac{p}{a} \xi,$$

and, further, let us divide R into two parts, M and N , such that

$$R = M + N,$$

and that

$$\frac{d^2 (\rho)}{dv^2} + (1 - \beta + X) (\rho) = M$$

$$\frac{d^2}{dv^2} \left(\frac{p}{a} \xi \right) + (1 - \beta + X) \frac{p}{a} \xi = N.$$

Since the functions M and N are still quite arbitrary, we can, in choosing them, satisfy a certain condition. The object of this condition is generally that one of the functions shall only contain terms of a certain form, whilst the other either does not contain such terms at all, or only in such a way as to annihilate terms of the same form which arise in the function to be determined.

Before going further we shall adduce some general considerations on the form of the integrals of a linear equation of the second order. Let the equation be

$$\frac{d^2y}{dv^2} + Xy = R. \quad (1)$$

We assume that the particular integrals of the equation

$$\frac{d^2y}{dv^2} + Xy = 0. \quad (2)$$

are known. If these integrals are denoted by y_1 and y_2 , and the constants of integration by g and h , we have

$$y = gy_1 + hy_2. \quad (3)$$

The functions y_1 and y_2 may now be multiplied by arbitrary constant factors, concerning which we say that they must be chosen so as to fulfil the equation

$$y_1y_2' - y_2y_1' = 1.$$

As is well known, the solution (3) may be extended so as to apply to the equation (1), if we regard the quantities g and h as functions of v instead of as constants. These functions, however, fulfil their object if they satisfy only a single equation of condition, which we shall give immediately; hence they may be chosen in any infinite number of different ways, without causing equation (3) to cease to give the integral of equation (1). By differentiation of (3) we obtain

$$y' = gy_1' + hy_2' + y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv},$$

whence

$$y'' = gy_1'' + hy_2'' + y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv} + \frac{d}{dv} \left(y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv} \right).$$

If we add this to equation (3), after it has been multiplied by X , we obtain, by help of (1) and (2), the following:—

$$y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv} + \frac{d}{dv} \left(y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv} \right) = R. \quad (4)$$

and this is the single equation of condition, which must necessarily be satisfied, in order that the value (3) of y may give us the complete integral of (1). The second condition, which must be assumed in order to obtain an actual determination of g and h , may remain entirely arbitrary.

If we take as this second condition—

$$y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv} = 0,$$

we obtain—

$$y' = gy_1' + hy_2'.$$

In this case not only has the integral of equation (1) the same form as that of equation (2), but the same is also true of the first differential coefficients. We may understand the results of this determination by a very simple example.

Let us take—

$$X=1; \quad R=\gamma \cos [(\mathbf{I}-\sigma)v-B],$$

where γ and σ are small quantities of the order of the disturbing forces, where γ further involves the excentricity as a factor, and where the quantity B denotes an arbitrary constant.

Then it easily follows that—

$$y_1 = \cos v; \quad y_2 = \sin v.$$

When these values are substituted in the formulæ—

$$\frac{dg}{dn} = -y_2 R; \quad \frac{dh}{dn} = y_1 R$$

they lead to the following expressions:—

$$\frac{dg}{dv} = -\frac{1}{2}\gamma\{\sin(\sigma v + B) + \sin[(2 - \sigma)v - B]\}$$

$$\frac{dh}{dn} = \frac{1}{2}\gamma\{\cos(\sigma v + B) + \cos[(2 - \sigma)v - B]\}$$

It may be remarked that g and h contain terms of two kinds; the first is an elementary term of the form (A), the second, on the other hand, does not belong either to the form (A) or to the form (B). This circumstance involves an unnecessary complication of the functions g and h , which may become extremely troublesome when we have to deal with continued approximations. The complication may, however, be avoided if we do not make the condition at our disposal as before, but make it so as to simplify the functions g and h . I remark that by such a simplification the meaning of the function y is unaltered, and only the meaning of the differential of y is affected. The loss of simplicity in the meaning of this differential is, however, in many cases (for example, in the planetary theory) a very slight evil in comparison with the advantage which is gained by the simplification of the functions g and h . Besides, the adoption of the above condition depends actually on the fact that this choice appears obvious, and that there is no other before us; it depends also on the circumstance that the Keplerian representation of the planetary motions can be retained unchanged, so that the disturbances may be regarded as small corrections which will exercise no characteristic influence on the representation of the motion.

We will now, however, set aside the condition chosen above, and will replace it by a more general one.

We write then—

$$y_1 \frac{dg}{dn} + y_2 \frac{dh}{dn} = -(\lambda) \quad (5)$$

so that (λ) is an arbitrary function of v . From equation (4) we then find—

$$y_1 \frac{dg}{dv} + y_2 \frac{dh}{dv} = R + \frac{d(\lambda)}{dv},$$

whence there results—

$$\left. \begin{aligned} \frac{dg}{dv} &= -y_2 \left(R + \frac{d(\lambda)}{dv} \right) - y_2'(\lambda) \\ \frac{dh}{dv} &= y_1 \left(R + \frac{d(\lambda)}{dv} \right) + y_1'(\lambda) \end{aligned} \right\} \dots \dots \dots (6)$$

Before going further we will prove directly that the function y may be made independent of the function (λ) by the substitution of the values of g and h , which result from the equations just found, in the equation (1).

For since—

$$\frac{d}{dv} (y_2(\lambda)) = y_2 \frac{d(\lambda)}{dv} + y_2'(\lambda)$$

$$\frac{d}{dv} (y_1(\lambda)) = y_1 \frac{d(\lambda)}{dv} + y_1'(\lambda),$$

we obtain—

$$g = g_0 - y_2(\lambda) - \int y_2 R dv$$

$$h = h_0 + y_1(\lambda) + \int y_1 R dv,$$

where g_0 and h_0 are constants of integration.

Hence, from equation (3) we obtain—

$$y = y_1 \left(g_0 - \int y_2 R dv \right) + y_2 \left(h_0 + \int y_1 R dv \right).$$

This is a value which is completely independent of the function (λ) , and hence it is the same which we should have found if we had made $(\lambda) = 0$.

As we have full disposal over the function (λ) , we can choose it so that the functions g and h may be as simple as possible; for example, they may be made to contain only elementary terms of the form (A), and this is especially advantageous in the planetary theory. In every such definition the equation of condition (4) must, of course, be satisfied, so that it is not possible completely to attain the intended object for every value of R . As already remarked, the differential equation must, for this reason, be divided into two, and the intended object, that the functions g and h may have the forms referred to, must be kept in view. And, further, certain terms of the function R must be retained whilst the others are carried over to the second equation. In the simple example considered above the matter is as follows:—the function R has already the form corresponding to our intention, for we may write—

$$R = \Psi \cos v + \Phi \sin v$$

where—

$$\Psi = \gamma \cos(\sigma v + B); \quad \Phi = \gamma \sin(\sigma v + B).$$

If now the values $y_1 = \cos v$, $y_2 = \sin v$ are substituted in the equation of condition (4), that equation can be satisfied by equating to zero the coefficients of $\cos v$ and $\sin v$ separately.

Hence, there result the equations—

$$\frac{d^2 g}{dv^2} + 2 \frac{dh}{dv} = \Psi$$

$$\frac{d^2 h}{dv^2} - 2 \frac{dg}{dv} = \Phi$$

This system can be integrated very easily. For if, after multiplying the second by $i = \sqrt{-1}$, we add the two equations together, we obtain the following linear differential equation of the second order:—

$$\frac{d^2}{dv^2}(g + ih) - 2i \frac{d}{dv}(g + ih) = \Psi + i\Phi,$$

whence it follows that

$$\frac{d}{dv}(g + ih) = e^{2iv} \int (\Psi + i\Phi) e^{-2iv} dv.$$

From this formula we may compute $\frac{dg}{dv}$ and $\frac{dh}{dv}$, whence g and h may be found by a simple quadrature.

We remark that the process remains unaltered when the functions Ψ and Φ retain any number of terms of the same form as those written above.

We now assume a somewhat more general form for the particular integrals y_1 and y_2 , writing

$$y_1 = f(v) \cos v; \quad y_2 = f(v) \sin v,$$

where f denotes an elementary function of the form (A). If now the form of the function R be assumed to be the same as in the preceding case, and if the coefficients of $\cos v$ and $\sin v$ be separately equated to zero, we find from equation (4)

$$f(v) \frac{d^2 g}{dv^2} + 2f(v) \frac{dh}{dv} + 2f'(v) \frac{dg}{dv} = \Psi$$

$$f(v) \frac{d^2 h}{dv^2} - 2f(v) \frac{dg}{dv} + 2f'(v) \frac{dh}{dv} = \Phi.$$

Whence

$$\frac{d^2}{dv^2}(g + ih) - (2i - 2 \frac{f'(v)}{f(v)}) \frac{d}{dv}(g + ih) = \frac{1}{f(v)} (\Psi + i\Phi).$$

From the integration of this equation we obtain

$$\frac{d}{dv}(g + ih) = \frac{e^{2iv}}{f(v)} \int f(v) e^{-2iv} (\Psi + i\Phi) dv.$$

With this result our problem may be regarded as solved, for when $f(v)$, Ψ , and Φ are given, there is no formal difficulty in obtaining the expressions for $\frac{dg}{dv}$ and $\frac{dh}{dv}$, and hence also for g and h .

We will now make a third assumption as to y_1 and y_2 , the basis of an investigation similar to that carried out above. We shall now, however, go somewhat further, and take into consideration the function which was previously regarded as indeterminate. This function has an actual application in the planetary theory.

Suppose, then, that

$$y_1 = f(v) \cos(v + \theta); \quad y_2 = f(v) \sin(v + \theta),$$

where f has the same meaning as before, and θ denotes a function of v which, besides one term multiplied by v , only contains elementary terms of the form (A).

On assuming for R the same form as above we obtain, by comparison of the coefficients of $\cos v$ and $\sin v$ in the equation of condition (4), the results

$$\begin{aligned} f(v) \cos \theta \frac{d^2 g}{dv^2} + f(v) \sin \theta \frac{d^2 h}{dv^2} + 2[\sin \theta f'(v) + \left(1 + \frac{d\theta}{dv}\right) f(v) \cos \theta] \frac{dh}{dv} \\ + 2[\cos \theta f'(v) - \left(1 + \frac{d\theta}{dv}\right) f(v) \sin \theta] \frac{dg}{dv} = \Psi \\ - f(v) \sin \theta \frac{d^2 g}{dv^2} + f(v) \cos \theta \frac{d^2 h}{dv^2} + 2[\cos \theta f'(v) - \left(1 + \frac{d\theta}{dv}\right) f(v) \sin \theta] \frac{dh}{dv} \\ - 2[\sin \theta f'(v) + \left(1 + \frac{d\theta}{dv}\right) f(v) \cos \theta] \frac{dg}{dv} = \Phi \end{aligned}$$

Whence we easily obtain

$$\left. \begin{aligned} f(v) \frac{d^2 g}{dv^2} + 2\left(1 + \frac{d\theta}{dv}\right) f(v) \frac{dh}{dv} + 2f'(v) \frac{dg}{dv} &= \Psi \cos \theta - \Phi \sin \theta \\ f(v) \frac{d^2 h}{dv^2} - 2\left(1 + \frac{d\theta}{dv}\right) f(v) \frac{dg}{dv} + 2f'(v) \frac{dh}{dv} &= \Psi \sin \theta + \Phi \cos \theta \end{aligned} \right\} \quad \dots \quad (\alpha)$$

If the second of these equations be multiplied by i , it follows that

$$\frac{d^2}{dv^2}(g + ih) + \left\{ -2i\left(1 + \frac{d\theta}{dv}\right) + 2\frac{f'(v)}{f(v)} \right\} \frac{d}{dv}(g + ih) = \frac{1}{f(v)} \{ \Psi e^{i\theta} + i\Phi e^{i\theta} \}$$

or

$$\frac{d^2}{dv^2}(g - ih) + \left\{ 2\left(1 + \frac{d\theta}{dv}\right) + 2\frac{f'(v)}{f(v)} \right\} \frac{d}{dv}(g - ih) = \frac{1}{f(v)} \{ \Psi e^{-i\theta} - i\Phi e^{-i\theta} \}$$

The integration of the first of these equations leads to the result

$$\frac{d}{dv}(g + ih) = \frac{e^{2i(v+\theta)}}{[f(v)]^2} \int f(v) e^{-2iv-i\theta} [\Psi + i\Phi] dv,$$

and a similar result is obtained from the second. If, now, θ is a known function, $\frac{dg}{dv}$ and $\frac{dh}{dv}$ may be immediately computed, and afterwards g and h . In the case, however, which now for a time demands our attention, this assumption is not suitable. We shall, on the contrary, assume that the function θ depends

directly on the quantities g and h to be determined, and that the relation between the three quantities is as follows—

$$\theta = \beta_2 \int \eta^2 dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

where β_2 denotes a small constant of the order of the disturbing masses, and where we have put $g^2 + h^2 = \eta^2$.

We suppose that g and h are small quantities of the order of excentricities; and, with respect to the functions Ψ and Φ , we retain the previous assumption, namely—

$$\Psi = \gamma \cos (\sigma v + B), \Phi = \gamma \sin (\sigma v + B);$$

whence it follows that

$$\Psi \cos \theta - \Phi \sin \theta = \gamma \cos (\sigma v + \theta + B),$$

$$\Psi \sin \theta + \Phi \cos \theta = \gamma \sin (\sigma v + \theta + B).$$

Since, now, g and h must be determined as elementary functions of the form (A), we must assume that $\frac{d^2g}{dv^2}$ and $\frac{d^2h}{dv^2}$ belong to a higher order of magnitude than $\frac{dg}{dv}$ and $\frac{dh}{dv}$.

If we neglect in a first approximation the former as well as the products of $\frac{dg}{dv}$ and $\frac{dh'}{dv}$ with $\frac{d\theta}{dv}$, we obtain

$$\left. \begin{aligned} g &= g_0 - \frac{1}{2}\gamma \int \sin(\sigma v + \theta + B) dv \\ h &= h_0 + \frac{1}{2}\gamma \int \cos(\sigma v + \theta + B) dv \end{aligned} \right\} \dots \dots \dots (\gamma)$$

where g_0 and h_0 denote constants of integration, and the function f is put equal to unity.

In many cases the expressions for the functions g and h may be freed from the integral sign by the application of the process of partial integration. We cannot, however, always obtain convergent developments by these means; and where we have to deal with terms which undergo so great an increase by integration that they may imperil the stability of the system, we have to strike out a new path.

We obtain a method more appropriate for our object if we form the sum of the squares of the equation (γ), multiply this sum by β_2 , and, finally, differentiate the result. By referring to the equation (β) we then have

$$\begin{aligned} \frac{d^2\theta}{dv^2} = & -\beta_2\gamma g_0 \sin(\sigma v + \theta + B) + \beta_2\gamma h_0 \cos(\sigma v + \theta + B) \\ & + \frac{1}{4}\beta_2\gamma^2 \sin(\sigma v + \theta + B) \int \sin(\sigma v + \theta + B) dv \\ & + \frac{1}{4}\beta_2\gamma^2 \cos(\sigma v + \theta + B) \int \cos(\sigma v + \theta + B) dv \quad . \quad . \quad . \quad . \quad (8) \end{aligned}$$

If we regard the coefficient γ as sufficiently small, we have the right to neglect in the first approximation the sum of the last two terms in the above equation. There then remains

$$\frac{d^2\theta}{dv^2} = -\beta_2 \gamma g_0 \sin(\sigma v + \theta + B) + \beta_2 \gamma h_0 \cos(\sigma \theta + v + B).$$

We here put

$$g_0 = e_0 \cos \varpi_0, \quad h_0 = e_0 \sin \varpi_0,$$

and, further

$$2V = \sigma v + \theta + B - \varpi_0;$$

so that we obtain

$$\frac{d^2V}{dv^2} = -\beta_2 \gamma e_0 \sin V \cos V.$$

This equation may be integrated at once. The result is

$$\frac{dV}{dv} = \gamma_0 \sqrt{\left(1 - \frac{\beta_2 \gamma e_0}{\gamma_0^2} \sin^2 V\right)},$$

where we denote the constant of integration by γ_0^2 .

We write further

$$\frac{\beta_2 \gamma e_0}{\gamma_0^2} = k^2,$$

and setting aside the constant which arises in the integration in hand, we finally obtain

$$V = am \gamma_0 v, \text{ mod. } k.$$

From this there results therefore

$$\sigma v + \theta + B = 2 am \gamma_0 v + \varpi_0.$$

The second approximation is to be computed from this, the process being easily carried out by means of Lamé's differential equation. It may be remarked that a term here makes its appearance which contains v as a factor; it arises because we have neglected certain quantities in the original differential equations. In a rigorous treatment of these equations terms of this kind may always be avoided if they are not necessarily involved in the original differential equations.

The method explained above is also worthy of notice, because it is applicable when the quantities $\gamma \cos B$ and $\gamma \sin B$ are not constant, but are subelementary functions of the form (A).

In these cases the result is expressed by means of elliptic functions, with a variable modulus, which is in fact an elementary function of the form (A). The investigation of the differential equation which appears here I have treated in a memoir as yet unpublished. Dr. Harzer has, however, already made use of the most important part of it in his investigation of the absolute orbit of *Hecuba*.

In the application of the above method of investigation to the planetary theory γ_0 is a quantity of the order of the disturbing forces, and therefore of the same order as β_2 ; the constant

γ is of the same order as $\beta_2 e_0$. Hence it follows that the modulus is of the same order as e_0 —that is to say, of the order of the excentricities.

After this preface we resume the equation,

$$\frac{d^2(\rho)}{dv^2} + (1 - \beta - X)(\rho) = M \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where we assume, as to the functions X and M , that they give rise only to elementary or subelementary terms of the form (B) in (ρ) .

As in the following investigations we need not revert again to the function ξ , we may for brevity write ρ instead of (ρ) . In consequence of the assumption that the result shall only contain terms of the form (B), we have also an equation of the form—

$$\begin{aligned} \rho = & \kappa \cos [(1-s)v - \Gamma] \\ & + \kappa_1 \cos [(\sigma_1 - s)v - B_1] + \kappa_2 \cos [(\sigma_2 - s)v - B_2] + \dots \end{aligned}$$

where κ and Γ denote the two constants of integration. The remaining coefficients and angular constants depend partly on the elements of the orbits of the disturbing bodies, and, if they belong to terms of the higher class, partly also on κ and Γ , as well as on the elements which determine the position of the orbit.

We now write

$$\begin{aligned} g = \eta \cos (\varpi - \Gamma) = & \kappa + \kappa_1 \cos [(\sigma_1 - s)v + B_1 - \Gamma] \\ & + \kappa_2 \cos [(\sigma_2 - s)v + B_2 - \Gamma] \\ & + \dots \\ h = \eta \sin (\varpi - \Gamma) = & \kappa_1 \sin [(\sigma_1 - s)v + B_1 - \Gamma] \\ & + \kappa_2 \sin [(\sigma_2 - s)v + B_2 - \Gamma] \\ & + \dots \end{aligned}$$

and thence obtain the following expression for ρ .

$$\rho = \eta \cos [(1-s)v - \varpi]$$

If now we put

$$\frac{d\rho}{dv} = -\eta \sin [(1-s)v - \varpi] - (\lambda)$$

we have

$$-(\lambda) = s\eta \sin [(1-s)v - \varpi] + \cos [(1-s)v - \Gamma] \frac{dg}{dv} + \sin [(1-s)v - \Gamma] \frac{dh}{dv}.$$

In accordance with previous assumption, the functions X and M depend on ρ^2 ; they may besides contain terms multiplied by η^2 and its powers, without affecting the intended form of the function ρ . We now conceive these functions separated into two in the following way:—

$$\begin{aligned} X = & \beta_2 \eta^2 + X_0 + X_2 + \beta_4 \eta^2 \cos [2(1-s)v - 2\varpi] \\ M = & M_1 + M_3 \eta^2 + (M), \end{aligned}$$

The functions X_0 , X_2 , M_1 and M_3 are as follows—

$$X_0 = \Sigma \beta_i^{(0)} \cos(\sigma_i v + B^{(0)})$$

$$X_2 = \Sigma \beta_i^{(2)} \cos[(2 - \sigma_i)v - B_i^{(2)}]$$

$$M_1 = \Sigma \gamma_j^{(1)} \cos[(1 - \sigma_j)v - B_j^{(1)}]$$

$$M_3 = \Sigma \gamma_j^{(3)} \cos[(1 - \sigma_j)v - B_j^{(3)}]$$

where

$$\beta_i^{(0)}, \beta_i^{(2)}, \gamma_j^{(1)}, \gamma_j^{(3)}$$

denote constant coefficients of the order of the disturbing forces depending on the elements and on the indexes i and j .

It is hardly necessary to remark that the B 's are angular constants and the σ 's small quantities of the order of the disturbing force. The function M has obviously been added in order to annihilate non-elementary terms which may arise.

In the following operations the last term in X would exercise a very detrimental influence, because, in a later integration, it would contain the variable v as a factor. The term in question may, however, be at once easily eliminated. Let us write the above expression for X as follows—

$$X = (\beta_2 + \frac{1}{2}\beta_4)\eta^2 + X_0 + X_2 + \beta_4\eta^2[\cos[2(1-s)v - 2\omega] - \frac{1}{2}].$$

We remark that the last term may in general be omitted or joined to M , because on multiplication by $\rho = \eta \cos[(1-s)v - \omega]$, it can only give rise to non-elementary terms. Terms of the same nature which make their appearance later may be set aside in a similar manner, and usually by only a small alteration of the constant.

We shall now endeavour to liberate the already simplified differential equation from the function X_2 ; this may be done by replacing ρ by a new function, ρ_1 , and v by a new variable, u ; ρ_1 and u being defined by

$$\rho = \frac{\rho_1}{1 + \psi}, \quad dv = \frac{du}{(1 + \psi)^2}.$$

Let us further choose the, as yet, indeterminate function ψ so as to satisfy the equation

$$\frac{d^2}{dv^2}(1 + \psi) + (1 + \psi)\{1 - \beta - X + X_2 + \Delta X\} = \frac{1 - \beta - X}{(1 + \psi)^3}. \quad (8)$$

By equation (7) we then obtain the following result:—

$$\frac{d^2\rho}{du^2} + \{1 - \beta - (\beta_2 + \frac{1}{2}\beta_4)\eta^2 - X_0 + \Delta X\}\rho_1 = \frac{M}{(1 + \psi)^3}. \quad (9)$$

We shall be able to dispose later of the function ΔX , in such a way that ψ may contain no terms of the form (A).

In passing on to the integration of equation (9), we write for brevity

$$Q = \beta + (\beta_2 + \frac{1}{2}\beta_4)\eta^2 + X_0 - \Delta X,$$

and put

$$\rho_1 = \frac{E_1}{\sqrt{1-Q_1}}$$

in the equation

$$\frac{d^2 \rho_1}{du^2} + (1-Q)\rho_1 = \frac{M}{(1+\psi)^3}.$$

Thus, we obtain

$$\frac{d^2 E}{du^2} + \frac{1}{2} \frac{\frac{dQ_1}{du}}{1-Q_1} \frac{E}{du} + \left\{ 1-Q + \frac{1}{4} \frac{\frac{d^2 Q_1}{du^2}}{1-Q_1} + \frac{5}{16} \left(\frac{\frac{dQ_1}{du}}{1-Q_1} \right)^2 \right\} E = \frac{\sqrt{1-Q_1} M}{(1+\psi)^3}.$$

And if we define Q_1 by the equation

$$Q - \frac{1}{4} \frac{\frac{d^2 Q_1}{du^2}}{1-Q_1} - \frac{5}{16} \left(\frac{\frac{dQ_1}{du}}{1-Q_1} \right)^2 = Q_1,$$

as may be very easily carried out by successive approximations, we have

$$\frac{d^2 E}{du^2} + \frac{1}{2} \frac{\frac{dQ_1}{du}}{1-Q_1} \frac{dE}{du} + (1-Q_1)E = \frac{\sqrt{1-Q_1} M}{(1+\psi)^3} \quad \dots \quad (10)$$

The integral of this equation is known. When $M=0$, whilst c_1 and c_2 are constants of integration, the solution is

$$E = c_1 \cos \left[\int \sqrt{1-Q_1} du \right] + c_2 \sin \left[\int \sqrt{1-Q_1} du \right]$$

The particular integrals of the equation (9), except as regards a constant factor, always involve the supposition that

$$y_1 = \frac{\cos \left[\int \sqrt{1-Q_1} du \right]}{\sqrt{1-Q_1}}; \quad y_2 = \frac{\sin \left[\int \sqrt{1-Q_1} du \right]}{\sqrt{1-Q_1}}.$$

One sees by the light of the preceding indications how the complete integral of this equation must be developed. The steps which are still necessary for the solution of the differential equation for the function

$$\theta = \beta_2 \int \eta^2 dv$$

involve too many details to be here entered on.

If we look back at the expressions for the functions g and h , we easily perceive that the constant part of η^2 (which will be denoted by e_0^2) is made up in the following manner—

$$e_0^2 = \kappa^2 + \kappa_1^2 + \kappa_2^2 + \dots$$

By light of the preceding expressions we now easily perceive that the coefficient denoted by s , which represents the ratio of the mean motion of the apses to that of the body, is given approximately by the formula

$$s = \frac{1}{2} \{ \beta + (\beta_2 + \frac{1}{2} \beta_4) e_0^2 \}.$$

If now the quantity e_0 is of the order of the excentricities—and that this is the case may be seen from the known motion of the apses in the principal planets—none of the coefficients κ_1, κ_2 , &c., can become so large as to get out of their class. The number of the coefficients which belong to this class cannot be very great, because otherwise the quantity e_0 would increase beyond its indicated limit. These circumstances show, of course, that the series which we have obtained for g and h , and also for the function ρ , converge; but the evidence is not complete, and we can merely say that this convergence is highly probable. The series, in fact, arise from series proceeding by successive powers, and in these series single individual terms may be increased by very small divisors of integration. Hence, if we set aside those individual terms, they will either converge or diverge, like series proceeding by successive powers. But there is another circumstance, which seems to put the convergence beyond all doubt, namely, that the convergence of the series for the differential coefficients of the function ρ , with reference to the variables v or u , is as good as that of the development of the function itself.

The integration of equation (8) still remains, and for this we pursue the following method. We somewhat transform the equation, and write it as follows:—

$$\begin{aligned} \frac{d^2}{du^2} (1 + \psi) + \left\{ 1 - \beta - (\beta_2 + \frac{1}{2}\beta_4)e_0^2 \right\} \left\{ 1 + \psi - \frac{1}{(1 + \psi)^3} \right\} \\ = (1 + \psi) \left\{ (\beta_2 + \frac{1}{2}\beta_4)(\eta^2 - e_0^2) + X_0 - \Delta X \right\} \\ - \frac{1}{(1 + \psi)^3} \left\{ (\beta_2 + \frac{1}{2}\beta_4)(\eta^2 - e_0^2) + X_0 + X_2 \right\}. \end{aligned}$$

On the supposition that ψ is a small quantity, we develop part of the last term of the right-hand side in ascending powers of ψ . Then putting

$$\beta + (\beta_2 + \frac{1}{2}\beta_4)e_0^2 = p,$$

we have

$$\begin{aligned} \frac{d^2}{du^2} (1 + \psi) + (1 - p) \left\{ 1 + \psi - \frac{1}{(1 + \psi)^3} \right\} = - \frac{X_2}{(1 + \psi)^3} - \Delta X \\ + 4\psi \left\{ (\beta_2 + \frac{1}{2}\beta_4)(\eta^2 - e_0^2) + X_0 \right\} \\ - \psi \Delta X \dots \end{aligned}$$

Now this equation can be integrated by continued approximations. For this end I write,

$$\psi = \psi_0 + \psi_1,$$

and define ψ_0 by the equation

$$\frac{d^2}{du^2} (1 + \psi_0) + (1 - p) \left\{ 1 + \psi_0 - \frac{1}{(1 + \psi_0)^3} \right\} = - \frac{X_2}{(1 + \psi_0)^3} - T. \quad (II)$$

where T denotes function which is intended to transfer certain

terms to the equation next below, from which ψ_1 is determined. If we only write down the most important terms, this equation becomes

$$\frac{d^2\psi_1}{du^2} + (1-p)\{4 + 30\psi_0^2\}\psi_1 = T - \Delta X + 4\psi\{(\beta_2 + \frac{1}{4}\beta_1)(\eta^2 - e_0^2) + X_0\} + \dots \quad (12)$$

If the functions X_2 and T vanish the integral of equation (11) may be given in a simple form; for we then have

$$1 + \psi_0 = \frac{\sqrt{[1 - \mathfrak{S} \cos(2u \sqrt{1-p} - \Pi)]}}{\sqrt[3]{1 - \mathfrak{S}^2}},$$

where $\mathfrak{S}$ and Π denote the two constants of integration.

In the following developments I shall permit myself some abbreviations which, however, are quite unimportant in the final result. In the first place I simply equate the factor $\sqrt{1-p}$ to unity, whenever it is not seen to be multiplied by u .

The above value of $1 + \psi_0$ must now be substituted in equation (11); the quantities $\mathfrak{S}$ and Π must, however, no longer be regarded as constant, but must be so defined that the functions

$$\mathfrak{S} \cos \Pi = G; \quad \mathfrak{S} \sin \Pi = H,$$

may only contain elementary terms of the form (A). In order to fulfil this definition the function T must be chosen in a corresponding manner.

In consequence of our supposition with regard to the functions G , H , and $\mathfrak{S}^2$ we shall neglect their second differentials, and then obtain from equation (11)—

$$\begin{aligned} & \frac{3}{2} \frac{\sin w \frac{dG}{du} - \cos w \frac{dH}{du}}{\sqrt[3]{1 - \mathfrak{S}^2} \sqrt{1 - \mathfrak{S} \cos(w - \Pi)}} + \frac{3}{4} \frac{(\sin w \cdot G - \cos w \cdot H) \left(\cos w \frac{dG}{du} + \sin w \frac{dH}{du} \right)}{\sqrt[3]{1 - \mathfrak{S}^2} \{1 - \mathfrak{S} \cos(w - \Pi)\}^{\frac{3}{2}}} \\ & + \frac{3}{8} \frac{\sin w \cdot G - \cos w \cdot H}{\{1 - \mathfrak{S}^2\}^{\frac{3}{2}} \sqrt{1 - \mathfrak{S} \cos(w - \Pi)}} \frac{d\mathfrak{S}^2}{du} \\ & = - \frac{(1 - \mathfrak{S}^2)^{\frac{3}{2}}}{\{1 - \mathfrak{S} \cos(w - \Pi)\}^{\frac{3}{2}}} X_2 - T, \end{aligned}$$

where w is written instead of $2u \sqrt{1-p}$.

The functions G and H must now be so defined as only to contain terms of the form (A). This condition is fulfilled if in the above equation the coefficients of $\cos w$ and $\sin w$ are made to vanish separately, and if at the same time all terms which do not satisfy the above condition are, by means of T , made to vanish.

If now we put

$$X_2 = \Psi \cos w + \Phi \sin w,$$

where Ψ and Φ only involve subelementary terms of the second

class of the form (A), we obtain, as applicable to the first approximation, the equations

$$\frac{3}{2} \frac{dG}{du} + \frac{3}{8} \frac{d\mathfrak{S}^2}{du} G = -(1-\mathfrak{S}^2)\Phi$$

$$\frac{3}{2} \frac{dH}{du} + \frac{3}{8} \frac{d\mathfrak{S}^2}{du} H = (1-\mathfrak{S}^2)\Psi$$

whence

$$G = -\frac{2}{3} \sqrt{1-\mathfrak{S}^2} \int (1-\mathfrak{S}^2)^{\frac{3}{4}} \Phi du$$

$$H = \frac{2}{3} \sqrt{1-\mathfrak{S}^2} \int (1-\mathfrak{S}^2)^{\frac{3}{4}} \Psi du,$$

also

$$\begin{aligned} & \frac{3}{2} \frac{\left[\sin w \frac{dG}{du} - \cos w \frac{dH}{du} \right] \left[\cos w \cdot G + \sin w \cdot H \right]}{\{1-\mathfrak{S} \cos(w-\Pi)\}^{\frac{3}{2}}} \\ & - \frac{3}{2} \frac{\left[\cos w \frac{dG}{du} + \sin w \frac{dH}{du} \right] \left[\sin w \cdot G - \cos w \cdot H \right]}{\{1-\mathfrak{S} \cos(w-\Pi)\}^{\frac{3}{2}}} \\ & + \frac{3}{8} \frac{[\sin w \cdot G - \cos w \cdot H] [\cos w \cdot G + \sin w \cdot H]}{(1-\mathfrak{S}^2) \{1-\mathfrak{S} \cos(w-\Pi)\}^{\frac{3}{2}}} \\ & = \sqrt{1-\mathfrak{S}^2} \cdot T. \end{aligned}$$

It may, however, be easily perceived that, in the development of this expression for T , terms arise of a high class—at least of the sixth—which may be retained in G and H . There is no obstacle, however, to taking account of them in equation (12). It is indeed advantageous to take account, in the definition of ψ_1 , of terms of higher classes, which may become very large in G and H by integration, and to incorporate them in that function. That such a term in ψ_1 cannot become of any arbitrary magnitude may be shown in the following manner.

For the sake of simplicity, we assume that in the value

$$\psi_0 = \sqrt{\frac{8}{30}} \cdot \alpha \cos [(1-\sigma)w - A]$$

α and A denote constants. According to the preceding explanation α is an elementary coefficient of the second class, and consequently of the second order of excentricities, but is not multiplied by a factor of the order of the disturbing forces; σ , on the other hand, is, as before, of that order of magnitude.

If now this expression be substituted in equation (12) we obtain

$$\frac{d^2 \psi_1}{dw^2} + \{1 + \alpha^2 + \alpha^2 \cos [2(1-\sigma)w - 2A]\} \psi_1 = \frac{1}{4} T \quad \dots \quad (13)$$

For the integral of the equation found by putting T equal to zero, an approximate value may be given, which is not only quite sufficient for our purpose, but even corresponds better than the complete value, because it is possible to judge more directly

of the magnitude of the constants which enter in the approximate than in the exact form. The approximate solution is

$$\psi_1 = c_1 \frac{\cos[(1-s)w] - \mu \cos[2(1-\sigma)w - 2A - (1-s)w]}{1 + \nu \cos[2(1-\sigma)w - 2A]} + c_2 \frac{\sin[(1-s)w] + \mu \sin[2(1-\sigma)w - 2A - (1-s)w]}{1 + \nu \cos[2(1-\sigma)w - 2A]}.$$

The constants of integration are here denoted by c_1 and c_2 ; μ is a coefficient which is very nearly equal to unity; and ν , being $\frac{1}{8}\alpha^2$, is a very small coefficient; and finally s is given by the formula

$$s = \sigma \pm \sqrt{(\frac{3}{16}\alpha^4 - \alpha^2\sigma + \sigma^2)},$$

whence it is obvious that s is a quantity of the fourth order with respect to the excentricities.

From the above expression, and from the magnitude of the quantity s , we must conclude that a subelementary term of the sixth class in T will appear as a subelementary term of the second class in the function ψ_1 .

It would carry us too far into details to extend this investigation to the more general case, where $a \cos A$ and $a \sin A$ are elementary functions of the form (A), and we must therefore pass this over.

It is often possible, by a very simple transformation, to reduce within the proper limits, terms which seem at first as though they would become very large in integration. It follows thence that we almost always carry out the integrations, after having partially developed the function to be integrated. Corresponding to this development there is also a certain development of the divisors of integration, which is not always convergent, even when the development of the function to be treated was, before integration, very highly convergent. These obstacles may be avoided by carrying over terms which occur in the wrong place in one equation, to another where the divisor of integration may be determined more correctly.

It would, however, lead to no useful result if, for example, we should propose to treat all the terms of X_2 in this function ψ_1 , for the utility of equation (12) depends on the fact that the quantities G and H , and also ψ_0 , were found as elementary functions of the second class.

If now a term should exist in M which would be very largely increased in ρ_1 , the result must be regarded as inappropriate; but by carrying over this term to equation (12) we again obtain an appropriate result. The following steps are necessary for this end:—

Let the term in question be

$$\gamma \cos[(1-\sigma)v - B].$$

Since now

$$(1-\sigma)v - B = (2-\sigma-s)v - B - \Gamma - [(1-\sigma)v - \Gamma],$$

we have

$$\gamma \cos [(1-\sigma)v-B] = \gamma \cos 2W \cos V - \gamma \sin 2W \sin V,$$

where for brevity we write

$$V = [(1-s)v - \Gamma]$$

$$2W = (2-\sigma-s)v - B - \Gamma.$$

But

$$\rho = g \cos V + h \sin V$$

$$\frac{d\rho}{dv} = -g \sin V + h \cos V - (\lambda).$$

From which equations we easily deduce the following values:—

$$\eta^2 \cos V = g\rho + h \frac{d\rho}{dv} + h(\lambda)$$

$$\eta^2 \sin V = h\rho - g \frac{d\rho}{dv} - g(\lambda);$$

whence

$$\begin{aligned} \gamma \cos [(1-\sigma)v-B] &= \frac{\gamma}{\eta^2} \{ \cos 2W [g\rho + h \frac{d\rho}{dv} + h(\lambda)] \\ &\quad + \sin 2W [h\rho - g \frac{d\rho}{dv} - g(\lambda)] \}. \end{aligned}$$

From these results it is already obvious, without repeating the development, that the term in question will be carried over to the function ΔX ; for the term multiplied by $\frac{d\rho}{dv}$ may easily be eliminated. The term which is carried over will now appear in equation (12), and will be treated like the others. The result of this transposition depends actually both on the fact that the factor $\frac{1}{\eta^2}$ may be developed, and on the magnitude of the coefficient γ . The latter must belong at least to the third class, and must of course contain a factor of the order of the disturbing force.

We have above found expressions in which the variable v is replaced by the new variable u . Since, then, the functions X and M were originally expressed in terms of the variable v , it is necessary to reduce the arguments which depend on v to the variable u , and *vice versa*.

The equation

$$dv = \frac{du}{(1+\psi)^2}$$

serves as the basis for this operation.

We develop it as follows:—

$$dv = \frac{du}{(1+\psi_0)^2} - \frac{2\psi_1 du}{(1+\psi_0)^3} + \frac{3\psi_1^2 du}{(1+\psi_0)^4} - \dots$$

Since the even powers of ψ_1 contain constant terms, the non-periodic part of v has the form

$$(1+g)u$$

where g is a very small quantity, at least of the second order of the disturbing forces.

But the appearance of this constant would have been easily avoided if we had originally put

$$dv = \frac{(1+g)du}{(1+\psi)^2},$$

and had then determined g in such a way that the non-periodic part of the coefficient of du was equal to unity.

All of the terms, with the exception of the first by which dv is represented above, are multiplied by a factor of the order of the disturbing forces; they can, after both integrations, at most become elementary. For since ψ_0 is not itself multiplied by a factor of the order of the disturbing forces, we might conjecture that in the integration of the term $\frac{du}{(1+\psi_0)^2}$ hyper elementary terms would make their appearance. This is, however, not the case, as we may easily convince ourselves by substitution of the value

$$1 + \psi_0 = \frac{\sqrt{(1 - \beta \cos(w - \Pi))}}{\sqrt{(1 - \beta^2)}}.$$

If, in fact, we put $\beta = \frac{2a}{1+a^2}$, we have

$$\frac{du}{(1+\psi_0)^2} = \frac{(1-a^2)du}{1-2a \cos(w-\Pi) + a^2} = [1 + 2a \cos(w-\Pi) + 2a^2 \cos 2(w-\Pi) + \dots] du.$$

An inspection of this expression suffices to convince us of the absence of hyper elementary terms.

In the preceding pages a method of computing the radius vector in the absolute orbit has been laid down, which does not, indeed, appear at present entirely free from objection in every point, but is, at any rate, characterised by the advantage that by its difficulties may be avoided which previous methods (for example, that of Leverrier) were not in a position to overcome.

I cannot close this explanation without making a remark which bears on the conception of the absolute orbit, in contrast with the conception of the so-called mean ellipse—assuming this latter conception to be in general possible. If we substitute for the elements in the formulæ of elliptic motion those functions which represent the secular variations by trigonometrical functions, we obtain the expressions for the motion in the absolute orbit. This orbit is, however, not an ellipse, nor in this formula of motion is Lagrange's condition,* $(\lambda) = 0$, satisfied. An ellipse of which the elements are subjected only to secular inequalities, and which at the same time satisfies the above conditions, does not give what is required. The nomenclature 'mean ellipse' simply contains in itself a 'contradictio in adjecto.' The ellipse whose elements are subject to secular inequalities

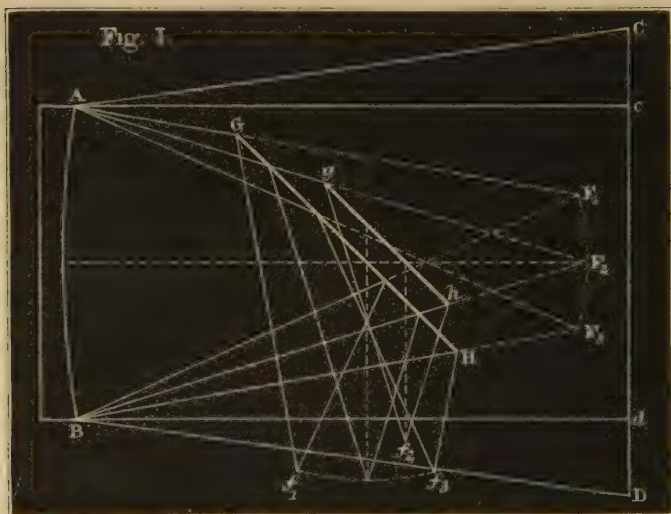
* In the variations of the planetary element the corresponding equation is written somewhat differently.

may indeed be employed as a means of construction for indicating the absolute orbit; but it intersects the latter at an angle of the order of the product of the excentricity and of the disturbing force, and this angle only vanishes when the radius vector coincides with an apse.

*Notes on Reflecting Telescopes, by Lieut.-Gen. J. F. Tennant,
R.E., F.R.S.*

In the course of discussions as to making Reflecting Telescopes, I have been led to investigate some formulæ which have not, so far as I know, been published, and which seem likely to be of use.

It has been usual in designing a reflector for astronomical purposes to determine the size of the small mirror by a formula which only gives it large enough to receive and reflect all the rays which come from an object in the axis of the great speculum; and the tube is only so much larger than the speculum as is necessary to allow the space for the cell. When, however, it is desired to use the telescope for photographic purposes, and a field of some diameter is sought, this plan no longer suffices. I propose in the first place to consider the necessary dimensions of the small mirror and tube. M. Lœwy has given an approximate formula in a paper on "A New Form of Reflector" in the *Bulletin Astronomique*.



In Fig. I. let AB be a section of the parabolic speculum, with its geometrical focus at F_2 , which will also be the focus of rays

parallel to the axis after reflection at the surface. So long as we only consider such rays, the mouth of the telescope need not be wider than cd , except by such amount as will suffice to make the rays quite clear of the edge, and a flat of the size gh in the position shown will reflect all the light to a focus at f_2 outside the tube. Denoting AB by $2r$ and the focal length by a , then it will only be necessary that c , the perpendicular distance of f_2 from the axis of the parabola, should be a little greater than half the diameter of the tube, and we shall have the following formula to determine the size of gh :—

$$\tan \phi = \frac{r}{a}; \quad m \text{ the minor axis of } gh = c \tan 2\phi \text{ and } l \text{ the major axis} = m\sqrt{2}.$$

If, however, the field subtends an angle $R_1AR_3 = L_1AL_1 = 2a$, we shall have to modify the design as follows:—

1st. The mouth of the telescope must have an enlarged diameter CD .

2nd. The image formed by the great speculum alone will now be enough to fill the space F_1F_3 , and we want to place it outside the tube in f_1f_3 , we must place the small mirror (or flat) nearer the great one and enlarge its dimensions: it will now be represented by GH . Using the same symbols as before, we now have

$$CD > 2(r + Ac \tan a) \text{ where } Ac \text{ is the length of the tube};$$

$$b = r \cot(\phi - \alpha); \quad m = (b - a + c) \tan(2\phi - \alpha) \text{ and } l = m\sqrt{2}.$$

If $\alpha = 0$ $b = r$, and the formula is reduced to that before given.

In order to show how much provision is necessary for a field of some diameter, I will take the case of a telescope of 30 feet focus and 60 inches aperture, in which

$$\tan \phi = \frac{1}{12},$$

and

$$\phi = 4^\circ 45' 49''.$$

If only the image of a central object were required, we might assume $c = 36$ inches, and we should have

$$m = 6.04 \text{ in.}; \quad l = 8.55 \text{ in.}$$

If a field of 1° diameter be required, c might be taken $= 37$ inches, and we should have

$$m = 11.70 \text{ in.}; \quad l = 16.55 \text{ in.}$$

If the field were 2° in diameter, we might retain 37 inches for c , but we should have

$$m = 17.58 \text{ in.}; \quad l = 24.86 \text{ in.}$$

For a field of 4° diameter we make $c = 39$ inches and thus should have

$$m = 29.08 \text{ in., and } l = 42.08 \text{ in.}$$

It is seen, that to obtain a field double the size of the Sun or Moon, it is necessary to nearly double the size of the small mirror, and to stop nearly four times the light which need be obstructed in order to define a star, and in all cases the centre of the flat will be out of the axis of the tube.

In the Cassegrain form of telescope, where the secondary speculum is placed between the great speculum and its focus, and an image is formed behind the great speculum, c of our formula would represent the distance from the principal focus to the face of the secondary mirror measured along the axis of the tube, and our formulæ would be

$$\tan \phi = \frac{r}{a},$$

and

$$b = r \cot (\phi - \alpha) \text{ as before;}$$

and then

$$d \text{ the diameter of the secondary mirror} = 2(b - a + c) \tan (\phi - \alpha).$$

The value to be assigned to c will depend upon the desired size of the image behind the speculum. If that is to be m times larger than what is due to the great speculum alone, and δ be the distance which this image is behind the face of the speculum, then we should have

$$c = \frac{a + \delta}{1 + m}.$$

Applying this to the case of a great speculum of the same dimensions as before, we shall find that when $m = 1$ and the image is 12 inches behind the reflecting surface, $c = 186$ inches; if $m = 2$, then $c = 124$; if $m = 5$, then $c = 62$.

for $m = 1$.	{	The image of a point requires	$d = 31.00$
		a field of 1° diameter „	$d = 34.00$
		„ 2° „ „	$d = 37.11$
		„ 4° „ „	$d = 44.20$.
for $m = 2$.	{	The image of a point requires	$d = 20.67$
		a field of 1° diameter „	$d = 24.76$
		„ 2° „ „	$d = 28.95$
		„ 4° „ „	$d = 37.21$.
for $m = 5$.	{	The image of a point requires	$d = 10.00$
		a field of 1° diameter „	$d = 15.27$
		„ 2° „ „	$d = 19.22$
		„ 4° „ „	$d = 30.07$.

Clearly, then, the size of the mirror obstructing the light would be very large, unless the focal image of the great speculum were considerably magnified, and the field very restricted; so that this form of telescope, however well adapted for the examination of small objects, is not suitable for photography.

The Gregorian form of telescope requires larger secondary mirrors than the Cassegrain for small objects, and in all cases a longer tube, which is a serious evil. Probably it will not be selected for use in any case, unless it should be found that the concave form of the small mirror is easier to produce than the convex one required for the Cassegrain. The appropriate formulæ in this case will be

$$c = \frac{a}{m-1}, \quad b = r \cot(\phi + \alpha),$$

$$d = (a + c - b) \tan(\phi + \alpha).$$

On the whole it would seem that for photographic purposes a large telescope would probably be best designed for a front view, with the sensitive surface facing the speculum, and no second reflection. In such a telescope as I have spoken of before, a field of 4° would be about 25 inches in diameter, whereas the small mirror even for a Newtonian would have 29 inches for its minor axis, and the difference (if a round plate were used) would be ample for the holder and exposing arrangements. If, for convenience, a plate of the usual form were used, then the Newtonian form of telescope would have a slight advantage if the field were very large; but not with a small field. Thus a field of 1° diameter would be 6.5 inches across, and the diagonal of a square plate which would include this field would be 9.2 inches, which is less than 11.7 inches required for the minor axis of the small mirror.

When the flat or small mirror is insufficiently large it will produce a field whose brightness diminishes rapidly in passing away from the proper limit; and, moreover, when either the small speculum or the mouth of the tube is too small, the portion of the great speculum used to produce the image of an object near the margin of the field will not be circular, and we may expect some falling off in definition.

I next proceed to examine the image formed by a parabolic speculum. Let Fig. II. be a section through the axis AX of the parabola, whose revolution round it generates the surface of the speculum, PA being the curve, of which A is the vertex and S the focus. Take the axis of the parabola for the axis of X, and A as the origin of rectangular co-ordinates.

Suppose three rays, L_1P_1 , LP_1 , and L_2P_1 , to be incident on a point P, whose co-ordinates are x' and y' . Then the reflected ray corresponding to LP_1 will pass through S. Let P_1Q and P_1q be the directions of the other two rays respectively. Call α the angle between P_1L and each of the other incident rays; then

it will also be the angle between PS , and either P_1Q or P_1q . Make $\tan \alpha = c$.

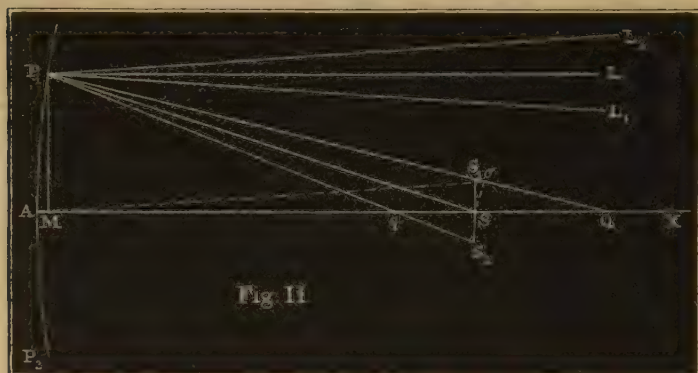


Fig II

Let S_1 be the focus for rays incident on P_1 and in the direction L_1P_1 , and let its co-ordinates be x_1, y_1 . Write v for $\frac{y'}{a}$, then $x' = \frac{av^2}{4}$. Also write h and k for $\frac{x}{a}, \frac{y}{a}$ respectively.

Then

$$\tan P_1SA = \frac{y'}{a-x'}$$

and the angle

$$P_1QA = P_1SA - \alpha;$$

therefore

$$\tan P_1QA = \frac{\frac{y'}{a-x'}}{1 + \frac{cy'}{a-x'}} = \frac{y' + cx' - ca}{a + cy' - x'}$$

and the equation to P_1S_1 will be

$$y - y' = \frac{ca - cx' - y'}{a - x' + cy'} (x - x'),$$

or

$$a(k-v) = \frac{ca - \frac{cav^2}{4} - av}{a - \frac{av^2}{4} + cav} \left(ah - \frac{av^2}{4} \right)$$

or

$$4(k-v) = \frac{4c - cv^2 - 4v}{4 - v^2 + 4cv} (4h - v^2)$$

Now differentiating with respect to v , h will disappear, and the value of h deduced from the result will be h_1 , the value of

h at S_1 . Performing this operation and changing the signs we get

$$4 = 2v \frac{4c - cv^2 - 4v}{4 - v^2 + 4cv} + (4h_1 - v^2) \frac{(4 + 2cv)(4 - v^2 + 4cv) + (4c - 2v)(4c - cv^2 - 4v)}{(4 - v^2 + 4cv)^2}$$

whence

$$4h_1 = v^2 + \frac{32 + 48cv + 16cv^2 + 8cv^3 + 2(2c^2 - 1)v^4 - cv^5}{2(1 + c^2)(4 + v^2)}$$

and

$$h_1 = \frac{8 + 12cv + 6c^2 8^2 - cv^3}{8(1 + c^2)},$$

or

$$x_1 = \frac{a}{8} \cos^2 \alpha \{8 + (12 - v^2)v \tan \alpha + 6v^2 \tan^2 \alpha\}$$

Evidently, if x_2 be the co-ordinate for a similar ray incident on P_2 opposite P_1 , we shall have

$$x_2 = \frac{a}{8} \cos^2 \alpha \{8 - (12 - v^2)v \tan \alpha + 6v^2 \tan^2 \alpha\}$$

by changing the sign of v .

Substituting the value of h_1 in the value found for $4k$ we shall get k_1 , the value of k at S_1 ,

$$k_1 = \frac{c}{8(1 + c^2)} \{8 + 12cv - 6v^2 - cv^3\},$$

or

$$y_1' = \frac{a}{8} \sin \alpha \cos \alpha \{8 + (12 - v^2)v \tan \alpha - 6v^2\}$$

and we shall have

$$y_2 = \frac{a}{8} \sin \alpha \cos \alpha \{8 - (12 + v^2)v \tan \alpha - 6v^2\}$$

also

$$x_1 - x_2 = \frac{av}{4} \cos \alpha \sin \alpha (12 - v^2),$$

and

$$y_1 - y_2 = \frac{av}{4} \sin^2 \alpha (12 - v^2).$$

When an oblique ray is incident at the centre of the speculum, we shall have

$$v = 0$$

and

$$x_1 = x_2 = a \cos^2 \alpha, \quad y_1 = y_2 = a \cos \alpha \sin \alpha.$$

Hence the locus of the foci of such rays will be a spherical surface touching the paraboloid at its vertex, and passing

through the focus; its radius will be $\frac{a}{2}$,* and there will be no aberration.

In the case of a telescope of the dimensions I have spoken of we shall have

		$v = +\frac{1}{4}$ in.	$v = 0$ in.	$v = -\frac{1}{4}$ in.
Field 4° diameter	$x +$	361.1325	359.5615	357.9953
	$y +$	12.5455	12.5562	12.4360
Field 2° diameter	$x +$	360.3735	359.9981	359.5885
	$y -$	3.1449	3.1414	3.1380

The above applies to rays oblique to the axis of the telescope, and incident on small surfaces at any part of the speculum in the plane containing the axis and the incident ray. I now proceed to consider the case of a ray falling at a point out of this plane, and at a distance r from the axis. Calling the plane I have used the principal plane, let the line r make an angle with it $= \phi$.

Let x, y, z be co-ordinates whose origin is the vertex, and whose directions are: of x in the axis, of y perpendicular to it in the principal plane, and of z perpendicular both to the axis and principal plane; and let x_0, y_0, z_0 be the corresponding co-ordinates when the plane containing r and the axis is substituted for the principal plane. In this case the projection (on the plane XY_0) of the ray incident at the vertex will be inclined to the axis of X at an angle whose tangent is $c = \tan \alpha \cos \phi$.

Hence (making $v = \frac{r}{a}$) the equations of the reflected ray will be; in the plane X_0

$$y_0 - av = \frac{4c - 4v - cv^2}{4 + 4cv - v^2} \left(x - \frac{av^2}{4} \right)$$

or

$$y_0 = \frac{c(4 - v^2) - 4v}{4 - v^2 + 4cv} x + \frac{16 + 12cv + cv^3}{4 - v^2 + 4cv} \cdot \frac{av}{4}$$

and in the plane XZ_0

$$z_0 = \left(\frac{av^2}{4} - x \right) \tan \alpha \sin \phi.$$

* I am indebted to the Astronomer Royal for a very elegant geometrical demonstration of this property.

Through A draw $A\sigma$ parallel to AL_2 , and cutting P_1Q in σ . Then $A\sigma$ will be the reflected direction of the ray incident at A and parallel to L , P_1 , and the two angles $SP_1\sigma$ and $SA\sigma$ will each $= \alpha$. Hence a circle can be described through A, P_1, σ , and S . If P move towards A , σ will gradually move towards S_1 , and when P is at A will coincide with it, and in this limiting position the circle will touch the vertex of the parabola at A , and pass through S_1 and S .

Also since

$$y = y_0 \cos \phi - z_0 \sin \phi$$

and

$$z = y_0 \sin \phi + z_0 \cos \phi$$

we have

$$y = x \left\{ \frac{c(4-v^2)-4v}{4-v^2+4cv} \cos \phi + \tan \alpha \sin^2 \phi \right\} + \frac{av}{4} \left\{ \frac{16+12cv+cv^3}{4-v^2+4cv} \cos \phi - v \tan \alpha \sin^2 \phi \right\} \\ = \left(x - \frac{av^2}{4} \right) \tan \alpha + v \cos \phi \left\{ a + \left(av^2 - 4x \right) \frac{1 + \tan^2 \alpha \cos^2 \phi}{4 - v^2 + 4v \tan \alpha \cos \phi} \right\}$$

and

$$z = x \left\{ \frac{c(4-v^2)-4v}{4-v^2+4cv} \sin \phi - \tan \alpha \sin \phi \cos \phi \right\} \\ + \frac{av}{4} \left\{ \frac{16+12cv+cv^3}{4-v^2+4cv} \sin \phi + v \tan \alpha \sin \phi \cos \phi \right\} \\ = av \sin \phi \cdot \frac{(2 + v \tan \alpha \cos \phi)^2}{4 - v^2 + 4v \tan \alpha \cos \phi} - 4x \sin \phi \cdot \frac{v(1 + \tan^2 \alpha \cos^2 \phi)}{4 - v^2 + 4v \tan \alpha \cos \phi} \\ = av \sin \phi + v \sin \phi (av^2 - 4x) \frac{1 + \tan^2 \alpha \cos^2 \phi}{4 - v^2 + 4v \tan \alpha \cos \phi};$$

and if we make

$$R = av - v(4x - av^2) \frac{1 + \tan^2 \alpha \cos^2 \phi}{4 - v^2 + 4v \tan \alpha \cos \phi},$$

we shall have

$$y = \left(x - \frac{av^2}{4} \right) \tan \alpha + R \cos \phi \\ z = R \sin \phi.$$

If $v = 0$ we have $y = n \tan \alpha$, $z = 0$. This is the case of incidence at the vertex, and the reflected ray passes as it should through the focus in this case as before found.

If $\sin \phi = 0$ then z will always be 0. In this case, of course, the incident and reflected rays lie in the principal plane.

We see that—

1st. If we write $-\phi$ for ϕ , z will only change in sign, not in magnitude, while y will be unchanged. Therefore for any value of y there will be two equal values of z , with opposite signs, and the trace of the principal plane will divide the curve into two similar parts.

2nd. If $\sin \phi = 0$, then $z = 0$; but from value of R , we see that r and R will $= 0$ if

$$x = \frac{a}{4} \cdot \frac{(2 + v \tan \alpha \cos \phi)^2}{1 + \tan^2 \alpha \cos^2 \phi},$$

and we find that this will also be the case if

$$\cos \phi = \frac{2av \pm \sqrt{x(4a - 4x + av^2)}}{(4x - av^2) \tan \alpha}$$

This value of $\cos \phi$ becomes impossible if

$$x > a \left(1 + \frac{v^2}{4} \right).$$

When x exceeds this value, R cannot be $= 0$, but z will $= 0$ when $\sin \phi = 0$. Otherwise $\cos \phi$ may have two different values for which R and z vanish: these correspond to a point which is either double or quadruple.

It is needless to spend labour on seeking the place where the rays are most concentrated. The question of focus of the whole speculum in such a case seems to me likely to be affected a good deal by the causes which would make the image of a point of light, even in the case of a perfect reunion of the rays, to be a disc surrounded by rings.

I purpose adding some numerical results. As a circle, whose diameter is $1 \div \sqrt{2}$ of that of the speculum, will separate its surface into two zones, receiving equal amounts of light, I will only consider that ring as a mean case, and I shall assume $x = a \cos^2 \alpha$. In other respects I have used the dimensions I have taken before for the data of the telescope and calculated co-ordinates.

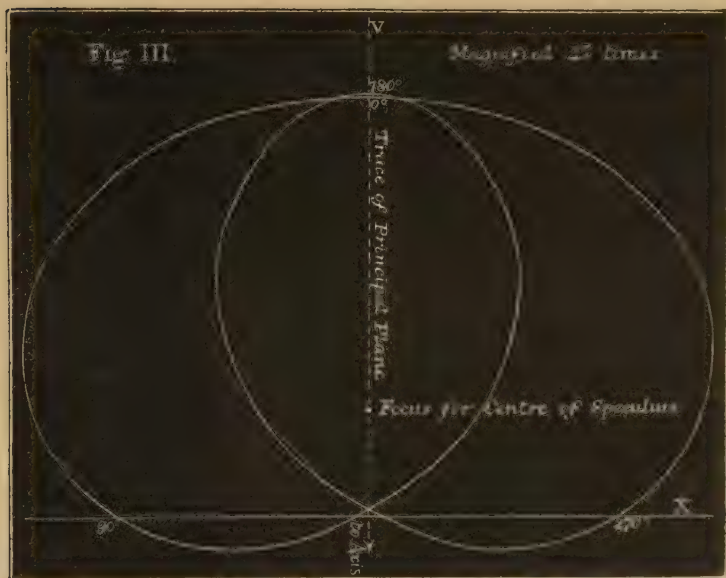
I shall assume α to be equal to 2° , or the field to be 4° in diameter, and from the values of y I shall omit $\left(x - \frac{av^2}{4} \right) \tan \alpha$; the whole of the co-ordinates will be expressed in thousandths of an inch.

When $R=0$, besides the two universal values of $\cos \phi \pm 1$, we shall only have one value of $\cos \phi$ possible, namely, that corresponding to $\phi = 117^\circ 42' 49'' \cdot 73$. And I may remark that only one such value of $\cos \phi$ will be possible when we are dealing with practical cases; because near, and at the focus, x will not differ much from a ; therefore the quantity under the radical sign in the numerator will not vary much from av , and the two values of the numerator will be, one very small, and the other not very far from $2av$; the denominator, on the other hand, will approach $4a \tan \alpha$ $\left(\frac{v^2}{4} \right.$ being small compared with 1), but $\tan \alpha$ will, for the outer part of the speculum, which alone is available, be much smaller than v ; and, of the two values of ϕ , one will be small as possible, the other greater than 1 .

Co-ordinates.			Co-ordinates.		
ϕ	$y - \left(x - \frac{av^2}{4}\right) \tan^2 \alpha$	z	ϕ	$y - \left(x - \frac{av^2}{4}\right) \tan^2 \alpha$	z
0	+43.62	0.00	90	- 0.00	+25.85
10	+42.99	+ 7.58	100	- 3.04	+17.45
20	+41.35	+15.04	110	- 2.70	+ 7.42
30	+38.31	+21.78	120	+ 1.22	- 1.13
40	+33.78	+28.86	130	+ 8.31	- 9.90
50	+27.76	+33.09	140	+17.47	-14.66
60	+20.60	+35.68	150	+27.20	-15.71
70	+12.91	+35.48	160	+35.79	-12.29
80	+ 5.67	+32.29	170	+41.74	- 7.31
90	+ 0.00	+25.85	180	+43.75	- 0.00

The co-ordinates for the values of ϕ from 180° to 360° will be those for $2\pi - \phi$ with the sign of z changed.

Of the portion of the value of y which I have omitted,



$x \tan \alpha$ is the value of y for centrally incident rays, and the remainder $\frac{av^2}{4} \tan \alpha$, equal to 10.96 thousandths of an inch, is the amount by which the focus of centrally incident rays is farther from the axis than the point from which the values in the table are reckoned: its cause will be evident if we observe

that $\frac{av^2}{4}$ is the portion of the axis of the parabola intercepted between the vertex, and the plane by whose intersection with the parabolic surface the reflecting ring is defined. I have plotted the curve in Fig. III. much enlarged, and shown on it the position of the focus of a centrally incident ray. The width perpendicular to the trace of the principal plane is 39.5 seconds of span.

When $x = a$ we shall have the case of the plane of the image being defined by its passing through the principal focus and being perpendicular to the axis. In this case

$$R = av \left\{ 1 - \left(1 - \frac{v^2}{4} \right) \frac{1 + \tan^2 \alpha \cos^2 \phi}{1 - \frac{v^2}{4} v + v \tan \alpha \cos \phi} \right\}$$

$$= av \tan \alpha \cos \phi \frac{1 - \frac{v^2}{4} - \tan \alpha \cos \phi}{1 + \frac{v^2}{4} \tan \alpha \cos \phi}$$

If we assume

$$\tan A = \frac{v}{1 - \frac{v^2}{4}}$$

and

$$\tan B = \tan \alpha \cos \phi,$$

we shall have

$$y = a \left(1 - \frac{v^2}{4} \right) \tan \alpha + av \cos \phi \tan B \tan (A - B)$$

$$z = av \sin \phi \tan B \tan (A - B)$$

Moving the origin to the point*

$$a \left(1 - \frac{v^2}{4} \right) \tan \alpha, 0$$

and using R and ϕ as the polar co-ordinates of the curve, we shall have

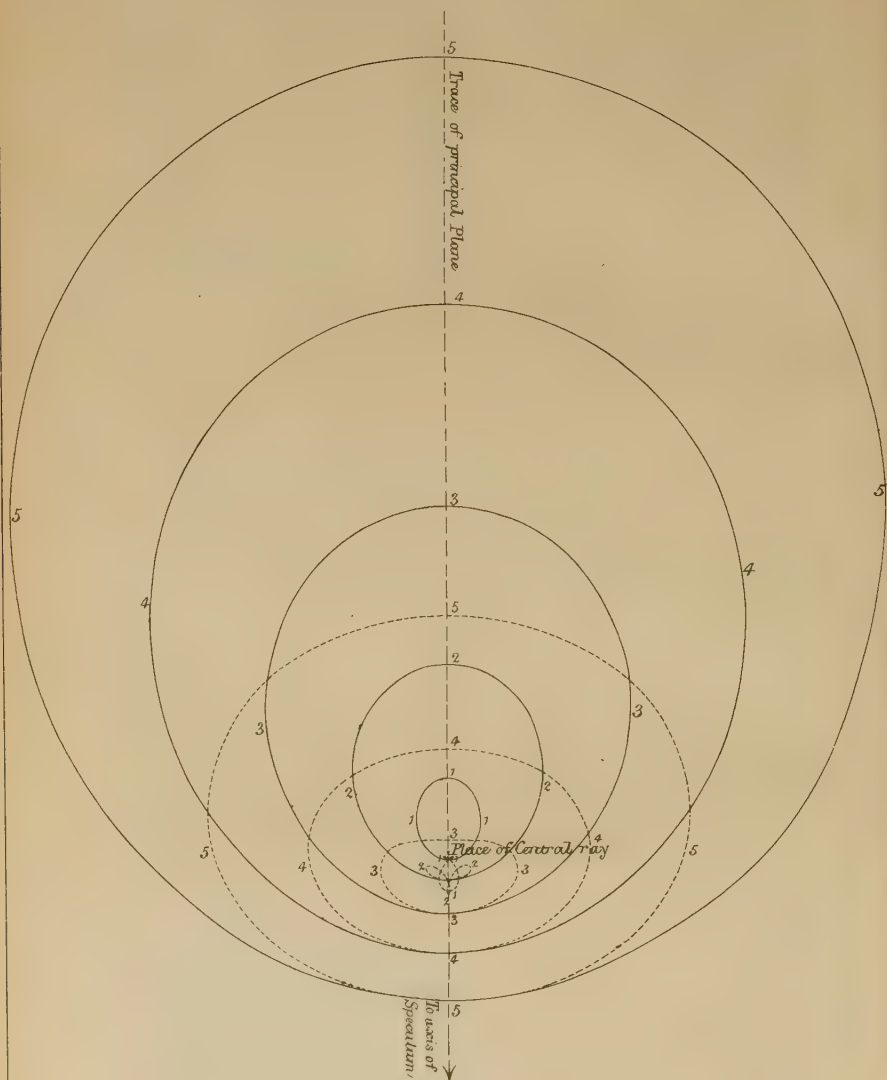
$$R = av \tan B \tan (A - B)$$

where A and B have the values above given, in which A depends only on v and B on α and ϕ .

In this case the values of $\cos \phi$ for $z = 0$ will be

$$+1, -1, 0, \text{ and } \left(1 - \frac{v^2}{4} \right) \tan \alpha$$

* If the plane of the reflecting ring were itself a reflecting plane, then the ray incident at its intersection with the axis would pass through this point.



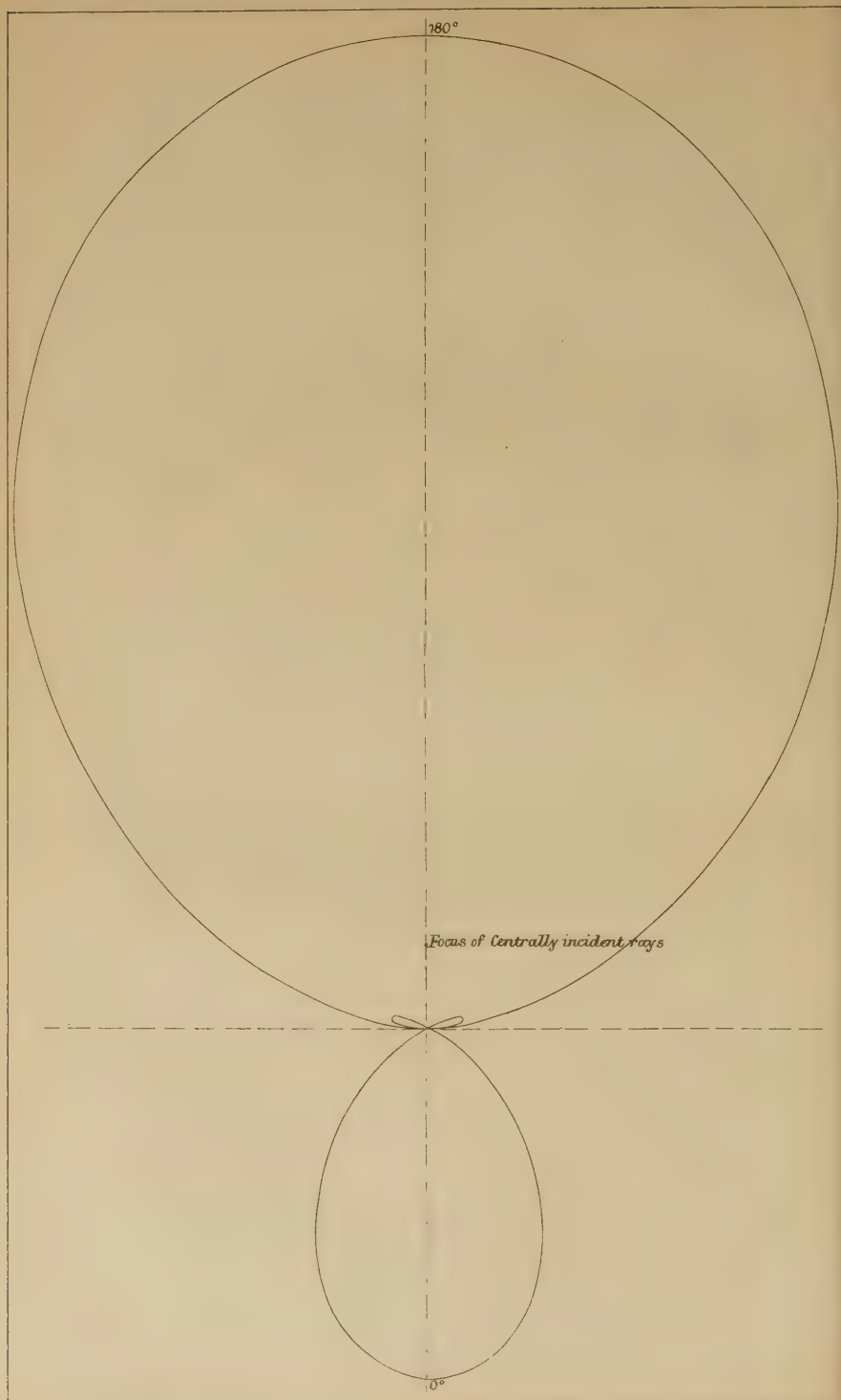
Length of Curve 1, solid line, $5''.00$; dotted line, $2''.69$; whole, $8''.10$

" " 2, " $15''.86$; " $2''.28$; " $16''.54$

" " 3, " $29''.62$; " $5''.25$; " $29''.62$

" " 4, " $47''.28$; " $14''.73$; " $47''.28$

" " 5, " $68''.86$; " $28''.08$; " $68''.86$



for both the last values y is reduced to the term

$$a\left(1 - \frac{v^2}{4}\right) \tan \alpha \text{ and } R=0,$$

hence the corresponding points will be quadruple if

$$\left(1 - \frac{v^2}{4}\right) \tan \alpha$$

be < 1 , and double it if it be > 1 .

This position would be that of most interest for photography, I have therefore computed the values of R for five equidistant rings of the speculum I have so often spoken of, and plotted them on a large scale in the annexed Plate 2* in which the dotted portion of each curve is due to the portion of the ring where $\cos \phi$ is positive, and the continuous portion to the remainder: α is assumed as 2° .

The place of the ray reflected at the vertex of the speculum is marked. It will be seen that as the ring increases in size the multiple point recedes towards the axis of the speculum; at first, while v is small, the curve is four-looped with two small loops; as v increases the loop towards the axis becomes smaller, while the others, and particularly the two side loops, increase in size. When

$$\tan \alpha = \frac{v}{1 - \frac{v^2}{4}}$$

the loop towards the axis vanishes, the multiple point is momentarily triple, and two of the branches form a cusp, the trace of the principal plane being a tangent to both branches. As v continues to increase, this cusp becomes a salient point, recedes from the axis, and gradually becomes effaced, the loops swell, and the curve gradually becomes one with only two rounded loops, with a common point in the axis.

We can form some idea now of the general distribution of the illumination, by picturing to ourselves a greatly increased number of these images from rings—the total image will be that of course arising from considering v to vary continuously from 0 to its maximum value. It would seem, that just behind the place of the ray reflected from the vertex, there will be a maximum of illumination in a bright ray extending from the place of the central ray towards the axis; from this the light will decrease. The lines of equal brightness will have a tendency to an oval form, and beyond this the light will spread out in the well-known fan shape.

If we consider what would be the result of photographing an object of this sort, I think it will be seen to be as follows. With comparatively short exposures the image will be elongated in

* Plate 1 shows the innermost curve on a larger scale.

lum is defined. Let S be the focus of the parabola, C the centre of curvature at the vertex, then $AC = 2a$. Also draw PM perpendicular to the axis AX, and PN the normal at P, then $MN = 2a$. And if PFf be the reflected ray (PC being incident) we shall have the angle $CPN = \text{angle } NPF$. I shall assume A as the origin of rectangular co-ordinates, and AX as the axis of X. $PM = y'$, $AM = x'$.

$$\frac{y'}{a} = v$$

as before, and consequently

$$x' = \frac{av^2}{4}$$

The angle

$$CPN = PCM - PNM$$

and

$$\tan PCM = \frac{y'}{2a - x'} = \frac{4v}{8 - v^2}$$

also

$$\tan PNM = \frac{y'}{2a} = \frac{v}{2}$$

Hence

$$\tan CPN = \frac{\frac{4v}{8 - v^2} - \frac{v}{2}}{1 + \frac{2v^2}{8 - v^2}} = \frac{v^3}{16 + 2v^2}$$

Whence

$$\tan AFP = (MNP - CPN) = \frac{\frac{v}{2} - \frac{v^3}{16 + 2v^2}}{1 + \frac{v^4}{32 + 4v^2}} = \frac{16v}{32 + 4v^2 + v^4}$$

The equation to PFf will therefore be

$$y - av = \frac{16v}{32 + 4v^2 + v^4} \left(\frac{av^2}{4} - x \right)$$

or

$$y = a \cdot \frac{32v + 8v^3 + v^5}{32 + 4v^2 + v^4} - x \cdot \frac{16v}{32 + 4v^2 + v^4}$$

When

$$y = 0 \quad x = a \cdot \frac{32 + 18v^2 + v^4}{16} = 2a + \frac{av^2}{2} + \frac{av^4}{16}$$

which value corresponds to F, and this will manifestly be the place of the bright image from the ring whose diameter is av or MP. Also

$$CF = \frac{av^2}{2} \left(1 + \frac{v^2}{8} \right)$$

Reverting to the equation

$$y = a \cdot \frac{32v + 8v^3 + v^5}{32 + 4v^2 + v^4} - x \cdot \frac{16v}{32 + 4v^2 + v^4}$$

and differentiating with effect to v we have

$$\frac{dy}{dv} = a \cdot \frac{1024 + 640v^2 + 96v^4 + 4v^6 + v^8}{(32 + 4v^2 + v^4)^2} - 16x \cdot \frac{32 - 4v^2 - 3v^4}{(32 + 4v^2 + v^4)^2}$$

and since when

$$\frac{dy}{dv} = 0, \quad x = Af \text{ (} f \text{ being the focus of the ray)}$$

we have

$$Ap = \frac{a}{16} = 2a + \frac{a}{16} \left\{ 24v^2 + 9v^4 + \frac{7}{2}v^6 + \&c. \right\} \cdot \frac{1024 + 640v^2 + 96v^4 + 4v^6 + v^8}{32 - 4v^2 - 3v^4}$$

and

$$Cp = a \left\{ \frac{3}{2}v^2 + \frac{9}{16}v^4 + \frac{7}{32}v^6 + \&c. \right\}$$

Also substituting the value of Ap for x in the equation for y we have

$$pf = -av^3 \cdot \frac{512 + 192v^2 + 32v^4 + 4v^6}{1024 - 80v^4 + 16v^6 - 3v^8} = -av^3 \left\{ \frac{1}{2} + \frac{3}{16}v^2 + \frac{9}{128}v^4 + \&c. \right\}$$

the values of Ap and pf being true to v^7 , which is more than enough.

These values of Cf , Ap and pf will serve in verifying the form of a speculum during the polishing process.

Data for Light Curves.

$$a = 360'000 \text{ inch; } \alpha = 2^\circ : 00'.$$

The units for the values of R is 0'001 inch.

No. 1. — $v = \frac{1}{80}$.

ϕ	R	ϕ	R	ϕ	R	ϕ	R	$\alpha \frac{v^2}{4} \tan \alpha$
0	— 3'82	50	— 0'78	100	— 0'83	150	— 8'52	
10	— 3'66	60	— 0'11	110	— 2'06	160	— 9'85	
20	— 3'25	70	+ 0'34	120	— 3'50	170	— 10'54	0'87
30	— 2'46	80	+ 0'39	130	— 5'27	180	— 10'82	
40	— 1'62	90	— 0'00	140	— 7'04			

No. 2. — $v = \frac{1}{30}$.

ϕ	R	ϕ	R	ϕ	R	ϕ	R	$\alpha \frac{v^2}{4} \tan \alpha$
0	— 1'24	50	+ 2'93	100	— 2'87	150	— 23'10	
10	— 0'43	60	+ 3'33	110	— 6'49	160	— 26'08	
20	+ 0'21	70	+ 3'07	120	— 10'65	170	— 27'98	3'49
30	+ 1'12	80	+ 1'98	130	— 15'08	180	— 28'64	
40	+ 2'11	90	0'00	140	— 19'31			

No. 3.— $v = \frac{1}{20}$.							
ϕ	R	ϕ	R	ϕ	R	ϕ	R
0	+ 9'49	50	+ 11'13	100	- 6'13	150	- 43'77
10	+ 9'67	60	+ 10'23	110	- 13'33	160	- 49'02
20	+ 10'16	70	+ 8'15	120	- 21'23	170	- 52'36
30	+ 10'76	80	+ 5'46	130	- 29'32	180	- 53'50
40	+ 11'20	90	+ 0'00	140	- 37'03		

No. 4.— $v = \frac{1}{15}$.							
ϕ	R	ϕ	R	ϕ	R	ϕ	R
0	+ 26'60	50	+ 23'83	100	- 10'56	150	- 70'53
10	+ 26'64	60	+ 20'63	110	- 22'57	160	- 78'58
20	+ 26'66	70	+ 15'70	120	- 35'33	170	- 83'66
30	+ 26'44	80	+ 8'83	130	- 48'12	180	- 85'40
40	+ 25'63	90	0'00	140	- 60'13		

No. 5.— $v = \frac{1}{12}$.							
ϕ	R	ϕ	R	ϕ	R	ϕ	R
0	+ 50'72	50	+ 41'02	100	- 16'28	150	- 103'44
10	+ 50'50	60	+ 34'53	110	- 34'22	160	- 114'80
20	+ 49'76	70	+ 25'60	120	- 52'95	170	- 121'93
30	+ 48'44	80	+ 14'08	130	- 71'46	180	- 124'40
40	+ 45'42	90	0'00	140	- 88'66		

P.S.—I have said that the devices for gaining a large field in achromatic combinations for photography are not available with the reflector. Of course there is no possibility of combining curves, and the other means is the use of a diaphragm to exclude from acting those parts of the surface which cause most diffusion. The position of this diaphragm should be so chosen that it flattens the field—that is, that the centre of the acting surface for oblique rays will have its focus in the plane perpendicular to the axis and passing through the principal focus.

Suppose we desire that this shall be the case for rays inclined to the axis. If we make x_1 in the formula giving the distance of this focus from the vertex = a , we shall have—

$$v^3 + 6v^2 \tan \alpha - 12v - 8 \tan \alpha = 0.$$

When α is small one root is very nearly $\frac{2}{3} \tan \alpha$, and can readily be determined; the other roots are large, one being positive, and the other negative. Practically $v = \frac{2}{3} \tan \alpha$ will be a sufficient approximation to the useful root. The place of the dia-

phragm will be $av \cot \alpha + a \frac{v^2}{4}$ (sensibly $\frac{2}{3}a$) from the vertex, and

this being nearly independent of α , it follows that the position flattens the focus for all obliquities. If D be the diameter of the

speculum, then that of the largest opening of the diaphragm will be

$$D - 2av = D - \frac{4a}{3} \tan a \text{ for the limiting value of } a.$$

If now we assume $a = 2^\circ$ we shall find the diameter of the diaphragm about $43\frac{1}{4}$ inches. The image for the field will be 25.14 inches in diameter, and probably the carriers will require to have a space of 27 inches in diameter allowed for them. This would leave only 28 per cent. of the surface available to form the image of any object; excluding all the central part.

On the Atmospheric Transmission of Visual and Photographically Active Light. By Captain Abney, R.E., F.R.S.

In his publication, 'Researches on Solar Heat,' which appears in the Professional Papers of the Signal Service, Professor S. P. Langley has been somewhat hard on those astronomers who have made a speciality of observing star-magnitudes. In the report on the transmissibility of our atmosphere for light, in which is embodied the observations made by his party at Mount Whitney, and which appears in the above-named volume, he makes the following remarks: 'For it may be observed in general terms that since the rays with large coefficients are represented by diminishing geometric progression, whose common ratio is near unity, these rays will persist whilst others with small coefficients are very nearly extinguished; and something like this was shown by Biot at the time when Melloni's first observations on the transmission of heat through successive strata attracted attention. But what we desire now further to point out is, that according as the difference of these coefficients of transmission for the different portions of the light of the same star is greater, so will the error of the result in treating them as equal be larger—a consequence so obvious that it is only necessary to make the statement in order to have its truth recognised.

'Since it has now been demonstrated that the formula ordinarily employed leads to too small results, it might properly be left to those who still employ it to show that their error is negligible; but this has never been done. There is possibly an impression that if there were any considerable error its results would become apparent in such numerous observations as have been made all over the world in stellar photometry during this century. But it is, in my opinion, a fallacy to think so; and I believe, as I have elsewhere tried to show, that the error might be enormous; that the actual absorption *might* be twice what it is customarily taken, or 40 per cent. instead of 20 per cent., without the error being detected by such observations as are now made.'

In a paper which I have recently communicated to the Royal Society, on the transmission of sunlight through our atmosphere, I

have described in detail observations which I have carried out during the past year on this subject, using the visible solar spectrum for the particular object I had in view. My wish was to ascertain the loss of luminosity of each visible ray after transmission through varying atmospheric thicknesses. The definite conclusion which I came to was that, as a rule, the loss of light followed Lord Rayleigh's law for scattering by small particles—

$$I' = Ie^{-kx\lambda^{-4}},$$

where I' and I are the transmitted and original intensities, x the thickness of air, and λ the wave-length. These observations which I made it is needless to describe again in this note; suffice it to say they were made by means of the colour photometer described by General Festing and myself last year in the Bakerian lecture. My standard luminosity curve was one taken at 8,500 feet altitude in the Alps, and, as I have already said, nearly every curve of luminosity through other air thicknesses obeyed the above law. Now, having got this result, it was easy to construct curves of luminosity of the spectrum for 1, 2, 3, . . . &c. air thickness in which the ordinates were the relative intensities of the rays transmitted. The areas of such curves when constructed could very easily be found, and such areas would represent the total luminosity of white light coming from the Sun though with varying thicknesses. It would then be easy to see what variation from the law usually employed in stellar observations would occur, and which, according to Langley, would be serious. The law in question is that—

$$I' = Ia^{\sec \theta},$$

I' and I being the transmitted and original intensities and a the coefficient of transmission, θ being the zenith distance, the secant of which, within certain limits, is a measure of the air thickness. I found that the minimum value of k in Lord Rayleigh's formula was .0013, when the value of λ^{-4} for $\lambda=6000$ was taken as 78.3. In this case the areas of the curves representing 0, 1, 2, 3, 4, and 5 atmospheres were represented as 761, 662, 577, 504, 439, and 385 respectively. It may be supposed that an observer when observing sunlight integrally would have obtained the same values, and then attempted to adapt the formula just given to it. Probably he would have used 'least squares,' and arrived at a result, but it would hardly have been necessary to do so. I had first omitted to calculate the area of the curve of the spectrum through 5 atmospheres, and I simply took 0 and 4 atmospheres as lying on the curve by the formula—

$$I' = Ie^{-\mu x},$$

where μ is the coefficient of absorption, and in which $e^{-\mu} = a$ in the above formula. From this I found that $\mu = .1378$, and this

gave for 1, 2, and 3 atmospheres 664, 578, and 504, the results from the spectrum being 662, 577, and 504. Calculating next the absorption through 5 atmospheres, I found it to be 383 instead of 385. The coefficient of transmission in the usual formula from the above is $\alpha = .862$, a somewhat high value; but it must be recollected it was my maximum value. Taking a larger value of k , viz. $k = .0019$, the areas for 0, 1, 2, 3, and 4 atmospheres were 917, 755, 623, 513, 418. Taking the first and last as lying on the logarithmic curve, the values obtained were 917, 755, 626, 508, 418, a very close approximation, μ being .19708, and the coefficient of transmission being .822. I may mention that with any value for k that I tried the same result held good. It thus appears that the logarithmic formula is more than a close approximation to the truth with the values arrived at from my observations.

Professor Langley's values and my own for the transmissibility of the different visible rays differ somewhat, so it occurred to me that it would be right to take his own values for transmissibility, and treat them in the same manner. Taking my Riffel observations of the luminosity of the spectrum as correct, I calculated from them the value of the areas of the curves of luminosity, using Langley's own coefficients of absorption. They were as follows, 1, 2, 3, 4, and 5 atmospheres, 657, 392, 235, 142, and 52. Taking 1 and 4 atmospheres as being on the logarithmic curve, the values derived from the formula were 657, 394, 237, 142, and 51, μ being .5106 and α being .602; a value which is extremely low.

This shows that still the agreement is complete between them. It must be borne in mind that in observations such as these, which are for settling stellar magnitudes, there is no desire to know the intensity of light outside our atmosphere. All that concerns the astronomer is to compare light coming from a star at the zenith with that coming from it at a reasonable zenith distance; 5 atmospheres is represented by a Z.D. of about $78^\circ 30'$, and it is unlikely that anyone would set much value on observations made at a greater Z.D. than that. Hence for the comparison of star magnitudes the usual formula may safely be adopted. A much more serious matter is the absorption coefficient used, and we find that it varies between .791 and .843, according to previous observations, whilst I have found my maximum at sea level to be .869, though my mean is not far from the .825, which value has been adopted by some astronomers.

The Z. distances of two observed stars may vary very greatly. This discrepancy of course would alter the calculated magnitudes very considerably, but if the difference be small the calculated magnitude will not be far out, even if the absorption coefficient be a little out from the truth. I may remark that in taking his stellar magnitudes, Professor Pritchard seems to have taken every care as to this, his comparison star being *Polaris*, and the star observed being about the same zenith distance.

Having arrived at the conclusion regarding visual observation, it struck me that absorption of the photographic spectrum ought to follow the same course. Now in the 'Proceedings of the Royal Society' I have given a very careful measurement of the photographic sensitiveness to various rays of light of bromo-iodide of silver, besides which I had the spectrum sensitiveness of other salts of silver by me, and I knew the air thickness and the spectral luminosity on the day when these measures were taken. Assuming that the ultra violet rays obeyed the same law as the visible rays, it was easy to calculate the photographic value of each ray for any air thickness; and this I accordingly did.

Using a value of $\cdot 00183$ for k in Lord Rayleigh's formula, the same process as before was gone through, supposing a plate of bromo-iodide of silver had been used: the areas were for 0, 1, 2, 3, and 4 atmospheres, 835, 621, 457, 340, and 254. The values arrived at by the logarithmic formula were 837, 621, 461, 342, and 254. This gave $\mu = \cdot 2980$, and the coefficient of transmission, $\cdot 742$.

Taking chloride of silver as the sensitive salt, its spectrum value being known, and proceeding as before, the following areas were found for 0, 1, 2, 3, and 4 atmospheres: 693, 413, 246, 147, and 87. Calculated by the logarithmic formula, they became 689, 413, 248, 148, and 87, $\mu = \cdot 5115$, and the coefficient of transmission $= \cdot 603$.

Taking these two results it is evident that before any definite knowledge of a value of a star magnitude by photography can be arrived at (supposing all stars were of the same colour) a definite acquaintance must be made with the kind of sensitive salt that is employed. Thus the bromo-iodide plate gave a value of transmission of $\cdot 742$; the chloride, of $\cdot 603$; whilst the optical value was only $\cdot 880$ when $k = 001183$.

Now it may be considered that these results are only theoretical deductions, but luckily they are capable of proof in the laboratory by experiment.

Water can easily be made turbid by alcoholic solutions of mastic or by other fine precipitates, and last year General Festing and myself proved that these artificially prepared turbid media strictly obeyed Lord Rayleigh's law. Now it was easy, having proved this, to ascertain whether when the light passing through such media was measured integrally by optical and photographic means the logarithmic formula was followed.

Experiment showed that it did so. Here is one of several results. A cell measuring exactly 6 inches by 4 inches was taken, and the values of light passing through these thicknesses of clear and turbid water were found to be—

				4-in.	6-in.
Clear	...	...	...	75.6	75.4
Turbid	...	...	...	26.5	15.5

Taking the clear or incident light as the mean of the two, 75·5, and using the 6 inches of the turbid medium as lying on the curve, the logarithmic formula gave $\mu = \cdot 2639$ in the formula

$$I' = Ie^{-\mu x},$$

and the absorption consequently for 4 inches of the fluid = 26·3.

Taking the photographic values a bromo-iodide plate was exposed to the clear, the 4-inch and the 6-inch turbid water for definite times, and the values of the light acting on the plate found to be

$$\begin{array}{rcl} \text{4-inch clear} & & 1 \\ \text{1-4-inch turbid} & & \frac{1}{10 \cdot 125} \\ \text{6-inch turbid} & & \frac{1}{33 \cdot 1} \end{array}$$

Using the logarithmic formula as before, taking the clear and 6-inch turbid light as lying on the curve, the value of the 4-inch turbid would be $\frac{1}{10 \cdot 33}$, and the value of $\mu = \cdot 5832$.

The optical value of μ is 2·639. The ratio of the optical to the photographic value of μ is 1 : 2·21, as determined by experiment.

Taking the optical and photographic values of μ , say, for a value of $k = 001183$, they are 1324 and 2980, or 1 : 2·25.

This shows, I think, that the values of μ determined experimentally in the laboratory and by appeal to the atmosphere are as consistent as can be expected.

Further we have this curious fact, that if we take the optical value of unanalysed light passing through the atmosphere, and also the photographic value of the same, we are able to form two equations which give the value of k , which must be applied to every wave-length, and also enable a curve of light to be constructed which takes into account general absorption by mist which is not composed of sufficiently fine particles to scatter light. Calculation shows that k is derived from either by dividing μ by 110 and 250 respectively when k is less than 0013. These gradually alter in value till they become 104 and 255, when $k = 0015$ is reached. The values of 110 and 255 are inverse fourth power of wave-lengths in the green and violet respectively, showing that when light is integrally measured it is equivalent to measuring definite rays in these parts of the spectrum.

I have not touched upon the question of how star magnitudes are to be measured, but merely ventured to give a means of applying a correction for loss caused by the atmospheric transmission. It will be seen how much greater is the loss of those rays which are photographically active than the visual rays, and it would appear that to enable a proper correction to be

made to photographs of stars taken at different times, the visual absorption at the time should also be calculated. I am not alluding now to the fact that the angle included in any star photograph is small; stars on such a plate may be compared *inter se*; but in order to make comparisons general the atmospheric absorption for the photographically active rays must be studied, and it is absolutely necessary that the spectrum value of the sensitive salt should be known beforehand.

Photographic Search for the Minor Planet Sappho.

By Isaac Roberts.

From 1872 to the present time no observations of the minor planet *Sappho* have been published except those of 1882, in which year Mr. Gill took measures for a determination of the solar parallax.

Mr. Bryant, who is engaged in determining the orbit of this planet, prepared an ephemeris for the opposition of this year, and in order that no time might be lost in identifying it from the neighbouring faint stars, and further that the error of the ephemeris might be obtained as early as possible, he sent, on December 16, 1886, the positions he had calculated, and appealed to me to find the planet if possible by photography.

The brightness of the planet is estimated at eleventh magnitude, and since its orbital movement in sixty minutes is equal to about 4.2 times its photographic diameter, the trail it would leave would probably not exceed in density a thirteenth magnitude star.

On December 30 I obtained, with an exposure of sixty minutes between sidereal time at Maghull, 7^h 35^m and 8^h 35^m, a negative of which the accompanying photograph marked chart No. 1 is an enlargement to three diameters. The trail of the planet is to be seen near the centre, and to make it more easily recognisable a white circle is drawn round it.

Another photograph was taken on January 1, and a third on the 14th. These are marked respectively charts Nos. 2 and 3. Each of the three charts shows the difficulties to be encountered in finding a planet so faint as this one amongst such numbers of other faint stars, and as an illustration I may refer to chart No. 3 upon which I estimate that there are about 2,000 stars below the seventh magnitude on each square degree.

Mr. Bryant informs me that the error of the ephemeris deduced from the photographs is in very close agreement with that from two meridian observations of the planet made about the same time at Dunecht, results that must be considered satisfactory.

This is probably the first instance in which photography has been successfully applied to the purpose here described, and as an historical fact it may be worth recording. There are also

some inferences which might be drawn from these experiments. First, it is here demonstrated that asteroids of eleventh magnitude leave very strong trails on the films of the photographic plates, and probably others down to the thirteenth or fourteenth magnitude could under favourable atmospheric conditions be photographed.

Another inference is that all the asteroids which up to the present time have been discovered, together with those that may exist but are not recorded down to the fourteenth magnitude, could by one astronomer alone be found and accurately charted in the course of two or three years' time.

On the Orbit of 14 (i) Orionis (O. Struve 98). By J. E. Gore.

I have computed the orbit of this binary star by means of the graphical method, and find the following provisional elements:—

Elements of 14 (i) Orionis.

$P = 190.48$ years	$\Omega = 99^\circ 35'$
$T = 1959.05$ A.D.	$\lambda = 302^\circ 42'$
$e = 0.2465$	$a = 1''.22$
$\gamma = 44^\circ 57'$	$\mu = -1.0.89$

From these elements we have the following formulæ:—

- (1) $u - 14.12 \sin u = -1.89(t - 1959.05)$
- (2) $\tan \frac{1}{2}V = 1.286 \tan \frac{1}{2}u.$
- (3) $\tan(\theta_c - 99^\circ 35') = 0.7077 \tan(V + 302^\circ 42')$
- (4) $\rho = 1.22(1 - 0.2465 \cos u) \cdot \frac{\cos(V + 302^\circ 42')}{\cos(\theta_c - 99^\circ 35')},$

where u is the excentric anomaly, and V the true anomaly for the epoch t ; θ_c the required position-angle, and ρ the distance.

The following is a comparison between the recorded measures and the positions computed from the above elements:—

Epoch.	Observer.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
		$^\circ$	$^\circ$	$^\circ$	"	"	"
1844.05	Mädler	258.8	254.44	+4.36	—	1.36	—
1844.53	O. Struve	250.83	253.95	-3.12	1.14	1.35	-0.21
1849.22	"	249.60	248.97	+0.63	0.98	1.33	-0.35
1852.15	Mädler	245.4	245.80	-0.40	—	—	—
1854.82	Dawes	240.9	242.75	-1.85	1.29	1.29	0.00
1859.22	O. Struve	237.80	237.55	+0.25	1.24	1.26	-0.02
1865.98	Dembowski	234.0	228.95	+5.05	1.25	1.20	+0.05

Epoch.	Observer.	θ_0	θ_c	$\theta^0 - \theta_c$	ρ_0	ρ^c	$\rho_0 - \rho^c$
		$^{\circ}$	$^{\circ}$	$^{\circ}$	"	"	"
1867·15	Dembowski	232·1	227·4	+4·70	1·28	1·19	+0·09
1868·14	"	228·5	226·1	+2·40	1·05	1·18	-0·13
1869·19	Dunér	224·6	224·63	-0·03	0·95	1·17	-0·22
1870·81	Dembowski	223·3	222·25	+1·05	1·21	1·16	+0·05
1870·87	O. Struve	224·13	222·17	+1·96	1·09	1·16	-0·07
1873·42	Dembowski	219·2	218·45	+0·75	1·26	1·14	+0·12
1876·18	Dunér	211·9	214·25	-2·35	1·22	1·11	+0·11
1876·88	Dembowski	212·8	213·17	-0·37	1·11	1·11	0·00
1877·18	Schiaparelli	209·9	212·71	-2·81	0·98	1·10	-0·12
1878·35	Dembowski	208·2	210·8	-2·60	0·99	1·10	-0·11
1879·10	Hall	205·8	209·61	-3·81	0·89	1·09	-0·20
1880·14	Jedrzejewicz	209·1	207·91	+1·19	1·03	1·08	-0·05
1881·13	Hall	203·2	206·28	-3·08	1·03	1·07	-0·04
1881·16	Jedrzejewicz	205·9	206·23	-0·33	1·08	1·07	+0·01
1882·14	"	201·3	204·62	-3·32	1·00	1·06	-0·06
1882·76	Engelmann	203·3	203·53	-0·23	1·15	1·06	+0·09
1886·917	Tarrant	196·4	196·15	+0·25	1·13	1·04	+0·09
1886·977	"	195·8	196·11	-0·31	1·16	1·04	+0·12
1887·019	Young	195·5	196·07	-0·57	0·98	1·04	-0·06

The magnitudes of the components are about 6 and 7. According to the above elements the distance will remain nearly constant during the next fifty years, the angle diminishing to about 107° in the year 1936. As far as I know the orbit has not been previously computed. Some of the measured angles in late years are very discordant. Professor Young's measure this year was communicated to me by private letter.

Observations of the Variable Star S (10) Sagittæ.

By J. E. Gore.

The following are my observations of this interesting variable star in the year 1886. They form a continuation of the observations given in the *Monthly Notices* for January 1886.

The comparison stars are, as before:—

					Mag.
11 <i>Sagittæ</i>	...	...	...	...	5·8
9 <i>Sagittæ</i>	...	...	...	...	6·6
DM + 16°, 4086	...	...	...	...	7·0

Observations of ι Sagittæ.

Date.	h	m	Mag.	Date.	h	m	Mag.
1886, Jan. 6	5	45	5.7	1886, Sept. 30	10	25	6.2
	13	6	0		30	11	20
May 19	10	25	6.1	Oct. 1	7	55	5.7
	21	10	50		1	10	40
	22	10	15		7	9	30
	26	10	30		9	7	10
	31	10	45		10	6	45
June 12	10	30	6.4		12	7	5
July 12	10	15	5.8		13	10	10
	19	10	0		16	7	15
	21	9	45		21	10	54
	22	10	10		23	6	45
Aug. 8	9	15	6.4		23	10	30
	14	9	0		24	6	26
	22	8	40		25	9	57
	28	9	0		26	6	37
	29	8	15		26	10	40
	31	8	20		28	7	7
Sept. 1	10	45	6.0		31	6	20
	2	8	25	Nov. 1	8	40	6.4
	3	8	20		3	6	56
	5	8	42		4	9	30
	6	7	58		5	7	50
	6	11	35		8	6	45
	8	8	25		10	8	0
	14	10	2		16	5	50
	15	7	40		17	5	43
	16	7	50		23	5	27
	17	7	55		26	5	30
	18	7	40		30	8	15
	18	11	20	Dec. 4	5	26	6.4
	20	8	10		12	6	10
	20	11	0		16	5	22
	22	7	10		22	5	4
	25	11	30		24	6	25
	27	7	5		25	5	30

A Proposed Nomenclature for Star Colours. By W. S. Franks.

It is generally admitted that the nomenclature of star colours is at present in a very unsatisfactory state, and not at all in keeping with the exact methods employed in other departments of astronomical observation. The hitherto popular fashion of naming star tints after fruit, flowers, precious stones, &c., is both vague and arbitrary, and also objectionable on account of the frequent diversity in tint of the object referred to. Any standard whatever, to be of real utility, must be readily accessible and also of uniform tint—the same at all times and in all places. It is very evident that the terrestrial objects just mentioned do not fulfil any one of these conditions; and so they should now be definitely rejected, as their retention only serves to perpetuate the existing confusion. The late Admiral Smyth, who was doubtless responsible for the introduction of many of these poetic but exceedingly vague terms, felt the force of the objections to their use, when he wrote the brochure entitled “*Sidereal Chromatics*”—a book which was unfortunately only intended for private circulation, and therefore not so well known as it deserves to be. It contained a plate of coloured discs, with four gradations each of red, orange, yellow, green, blue, and purple, which was intended as a scale upon which star colours could be more definitely expressed. But this chromatic diagram, pretty as it looked, had several serious faults. Its twenty-four tints were not so carefully selected as they might have been, nor, indeed, were they sufficient to meet the requirements of the observer; and further, an opaque wafer of colour, necessarily viewed by artificial light (which of course modified its appearance), could not well be compared with a glittering stellar point. The successive depths of tint, also, were inversely put with regard to the numerical order. Thus, the deepest tint of red was called “Red¹,” and the palest “Red⁴.” Some of the colours themselves are open to criticism. For instance, the two deeper shades of orange are not pure, but have a perceptible tinge of red, whilst the two corresponding shades of yellow are really orange-yellow. Neither the red nor the green are typical colours, as anyone may see by comparing them with the ordinary daylight spectrum; the former is too crimson, and the latter too bluish-green. There is some reference to the colours of stars in Professor Piazzi Smyth’s “*Madeira Spectroscopic*,” and a chromo-lithograph at the end of that work gives a graduated series of primary, secondary, and tertiary spectrum colours, with their corresponding wave-numbers.

It is pretty evident that, in the future, we must look to the spectrum for our standard colours. But as these star observations are confined to the period when the sun is below the horizon, it is equally obvious that they cannot be directly so

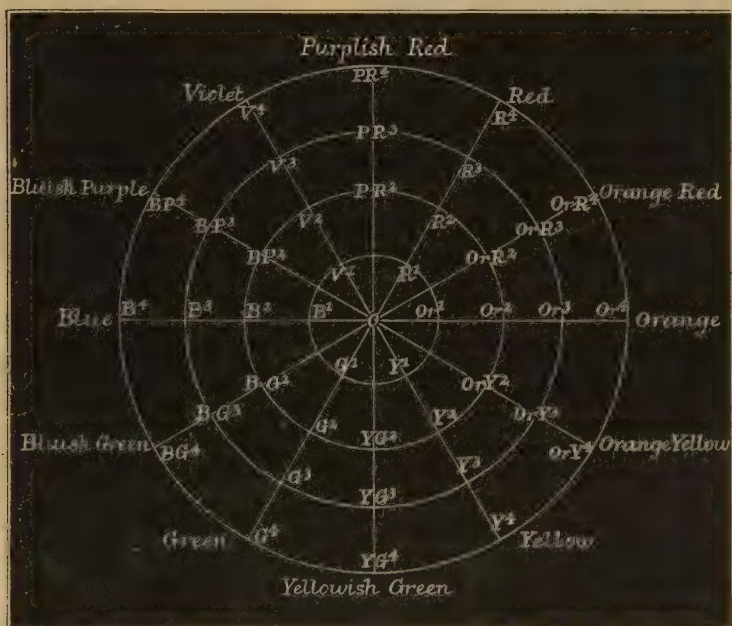
compared. This difficulty might be, however, got over by having a series of coloured gelatine discs carefully matched with the spectrum, and then viewing them as transparencies by the light transmitted from an incandescent electric lamp. In attempting to devise any plan for the more systematic observation of star colours, there are several essential points to be considered. The standard must be uniform, and easily accessible anywhere for reference, if it is to meet with universal adoption. The solar spectrum answers this end completely, since it affords a well-marked range of colours, which can be expressed in wavelengths, or referred to certain Fraunhofer lines. The nomenclature should be as simple as possible, yet copious enough for ordinary requirements, and be arranged on some methodical plan, so as to be easily remembered. Its terms should have a precise and definite meaning, and be capable of numerical reduction, so as to get mean results from a number of observations. How far these conditions are fulfilled in the scheme which I have the honour to lay before the Society may be best judged of by reference to the annexed diagram. The germ of the present plan is due to certain suggestions made by Professor Pickering, that I should reject all modifications of white, as applied to stars, except in the direction of the prismatic colours, and that I should take white as one end of a series for each colour, giving a numerical ratio to each term in the series. It then occurred to me to call white the zero, from which the spectral colours radiate as a centre, each concentric ring corresponding to a given depth of each colour. Starting with red, the primary and subordinate colours are placed at equal distances round the circle up to violet, and the opposite ends of the spectrum connected by the intermediate tint of purplish red. On this plan, too, each colour is opposite to its complementary; the sum of any two opposite colours forming white light. This may be convenient in estimating the colours of "double" stars. As to the fiducial points to which the chief colours are referred I would, provisionally, adopt Professor Smyth's definition of them. They would then run as follows:—

Red	...	...	* WN 36,000:	Nearest Solar lines a and B.
Orange	...	...	" 40,000:	" " α (alpha).
Yellow	...	...	" 43,000:	" " D ¹ and D ² .
Green	...	...	" 49,000:	" " b ¹ , b ² , &c.
Blue	...	...	" 56,000:	" " d.
Violet	...	...	" 60,000:	" " G and g.

With regard to the *names* of the colours, I have tried to avoid even such terms as "citron," "glaucous," &c., because, though used by Professor Smyth, they would certainly be misinterpreted by many observers. In adopting Professor Picker-

* The wave-number is for the *centre* of each colour.

ing's restriction of "white," I have sacrificed all such qualifying adjectives as very, intense, pale, greyish, &c., for, as generally used, these refer to the relative *brightness* of the light rather than to its colour. The terms I have proposed for the subordinate tints carry their own meaning, as they are compounded of the two adjacent primaries. And, by inverting the order, a minuter subdivision might be made, exactly analogous to the case of star magnitudes; as we could divide the interval between green and blue, say, into "bluish-green" and "greenish-blue," according to whether green or blue was the strongest component. But probably there would be no necessity for this, as the number of tints (forty-eight in all) seems sufficient for



the required purpose, and is twice as many as the number on Smyth's chromatic diagram. Commencing, now, with the centre as "white" (see diagram), the symbol for which is "0": the first circle will represent ruddy white, creamy white, yellowish white, greenish white, bluish white, and purplish white, their symbols being R^1 , Or^1 , Y^1 , G^1 , B^1 , and V^1 : the next circle *pale red*, *pale orange*, *pale yellow*, &c.; symbols, R^2 , Or^2 , Y^2 , &c.: the succeeding circle is intended to be equivalent to the *normal* shade of red, orange, yellow, &c.; symbols, R^3 , Or^3 , Y^3 , &c.: the outer circle corresponds to the deepest shades of colour, as *very red*, *very orange*, *very yellow*, &c.; symbols, R^4 , Or^4 , Y^4 , &c. Thus the four terms in each series

are supposed to be equal to four corresponding increments of depth of shade, the numerical ratios being *direct*, instead of *inverse*, as in Smyth's scale. And it is at once apparent how readily these terms admit of reduction in discussing a number of observations. I have not thought it necessary to specify the subordinate colours here, as their corresponding symbols appear on the diagram. But though they all follow the same invariable rule—that is, the *inner* circle means *white tinged* with the given colour: circle 2, a *pale* tint of the colour; circle 3, the *normal* colour; and circle 4, the *deepest* tint of colour, yet it will not, in all probability, be requisite to use the first term in their series, as six different tints of white are as many as can be well distinguished from each other.

P.S.—The original diagram, of which the annexed is a copy, was executed in water-colours, and gives a much better idea of the proposed scheme than one in black and white can possibly do, but it would probably be difficult and expensive to reproduce in a satisfactory manner.

On the Best Device for Revolving a Dome. By Professor
David P. Todd.

(Communicated by the Secretaries.)

The various methods which have been devised for applying power to revolve a dome show that it is a matter of considerable importance in observatory construction. The number of these methods shows also that many of them, at least, are imperfect either in principle or construction. A very common method, especially in the older observatories, consists of a continuous ratchet or cog-wheel run around the bed-plate or interior of the dome, in which a cog-wheel on a short journal runs, the power being communicated to this latter in the simplest manner. The difficulties in the way of this device consist rather in the imperfection of the running-gear of the dome itself. In constructing the large dome of the Washburn Observatory, at Madison, Wisconsin, the late Professor Watson sought to overcome these difficulties by a double cog-wheel, which should work into the ratchet of the dome on exactly opposite sides of the latter. In order to secure this end, a somewhat cumbersome apparatus was devised and applied, consisting of two vertical shafts on opposite sides of the dome geared into a rigid horizontal shaft, running across from one side to the other, underneath the floor of the dome. This apparatus is described in Vol. I. of the publications of this Observatory. In constructing the smaller dome of the Lick Observatory, the idea occurred to Mr. Fraser, the superintendent of construction, that the dome itself might be considered as a large pulley, and a rope run around it as such, and brought into the inside of the dome, where it was carried around a suitable

FIG. 1.

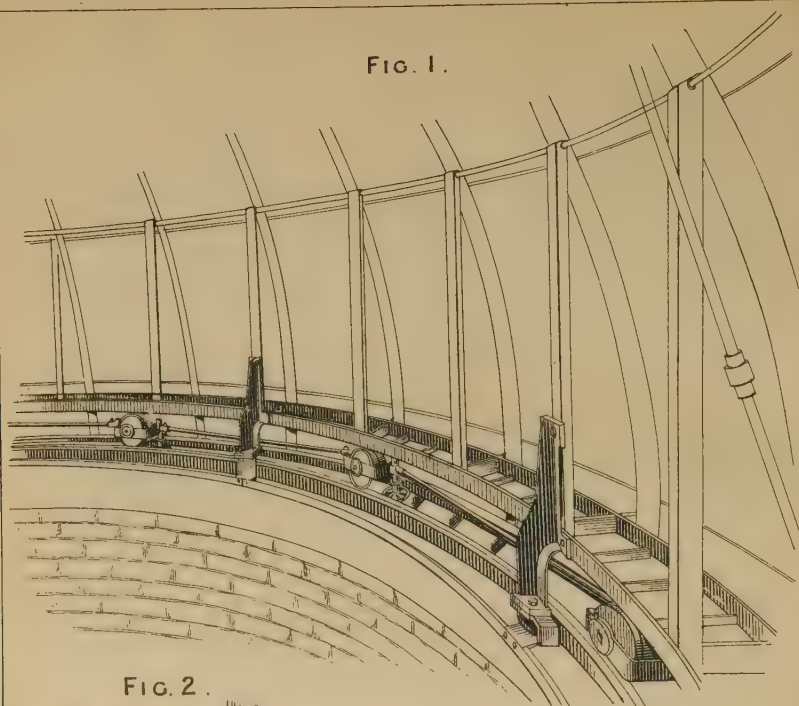


FIG. 2.

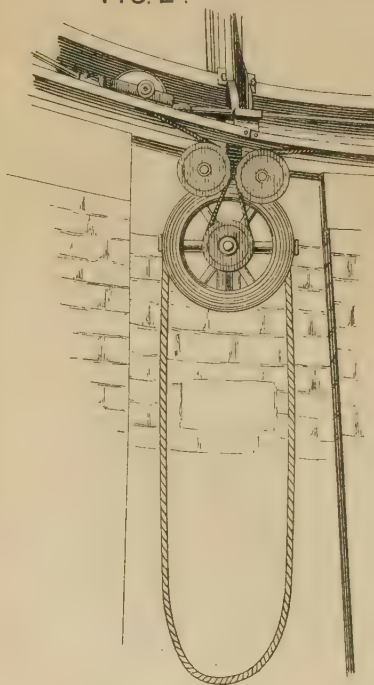
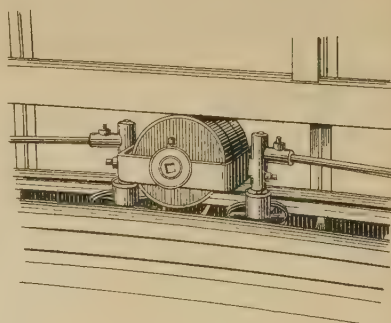


FIG. 3.



pulley connected with the gearing and a crank. This device works in the most perfect manner, and there is no objection to it except that, the rope being run around the outside of the dome, it is not easy to secure perfect weather protection at the point where the rope passes to the inside of the dome. In addition to this, the rope itself where exposed on the exterior of the dome would require protection from the weather, especially in our northern latitudes.

When Mr. Warner, of the firm of Warner and Swasey, of Cleveland, was at the Lick Observatory in May last, the idea occurred to him that the wire rope employed in turning the dome could, by means of a suitable device, be run around the inside of the dome quite as well as the outside, thus rendering special weather protection unnecessary, and reducing the apparatus to a very compact form. The accompanying plate (figs. 1-3) gives a good idea of this new device as constructed by Mr. Warner for the first time, and applied to the 22-feet dome of the new observatory at Smith College, Northampton. It will be seen that there are attached to the projecting uprights of the dome a series of horizontal grooved pieces, curved to the same radius as that of the dome itself. These pieces form a continuous grooved pulley on the inside of the dome, and concentric with it. Immediately underneath the bed-plate of the dome, and rigidly attached to the wall, is a framework carrying the pulleys as represented in the plate (fig. 2). Around the largest of these pulleys passes an inch rope, to which the necessary power is applied by the hand. On the same axle with this large pulley is a small pulley, around which a wire cable passes which revolves the dome directly. To facilitate the running of the wire rope from this small pulley into the large grooved pulley of the dome itself, a pair of idle pulleys are placed immediately underneath the grooved pulley attached to the dome. As these loose pulleys are placed very near together, they serve also the purpose of giving the rope a larger bearing service upon the motor-pulley below them. It will be seen that the device is a very simple and compact one. It is all contained in a shallow niche in the wall, and is thus entirely beyond the reach of the observing-chair as it moves upon the floor of the dome. The distribution of the power to every part of the circumference of the dome by means of the wire rope is an important advantage not secured in any form of cog-wheel apparatus. The bearing surface of the wire rope upon the dome is nearly 70 feet long; so that, although the rope may be quite loose, there is still friction enough to move the dome readily. The perfection of the running-gear of the dome itself has, of course, much to do with the ease of movement of this revolving apparatus. In the case of the dome alluded to above, a three-pound weight is sufficient to set the dome in motion, the ratio of descent of the weight to the movement of the periphery of the dome itself being about as three to one.

In connection with the accompanying illustration of a section of the running-gear of the dome, the following brief description will make its construction clear. First, being assured that the tracks on the wall-plate and the conical wheels are turned to their exact cones, the next important point is to hold these wheels so that their axes produced will converge to a point at the centre of the tracks. Messrs. Warner and Swasey have found it impossible to accomplish this by any system of bearing of the live ring. They therefore provide a yoke for each conical wheel. In addition to carrying the journals of the conical wheel, each yoke carries also two guide-wheels, which run between the tracks of the wall-plate. These guide-wheels are so adjusted as to keep the axis of the conical wheel coincident with the true radius of the circle of the wall-plate. These guide-wheels obviate also the necessity of flanges on the conical wheels, while the yokes merely require connection with each other by single flexible rods, forming thus the continuous live ring. By these simple devices the friction is so largely reduced that the dome may be readily revolved by the application of a direct tangential pressure of $1\frac{1}{2}$ pound for each ton's weight.

A Centering Tube for Reflecting Telescopes. By Edward Crossley.

To facilitate the centering of the "Common" 3-foot Reflector I have constructed a tube with diaphragms, by means of which the mirrors can be adjusted with the greatest ease.

The tube I use is 10 inches long and $2\frac{5}{8}$ inches in diameter. This is placed in the eye end at the side of the telescope. The end of the tube towards the eye has a small pin-hole. The other end towards the mirrors has two diaphragms which will replace each other concentrically. The large diaphragm is 1.96 inch in diameter, the small one is 0.37 in diameter. The large diaphragm just embraces the large mirror, which can be centered by means of the flat. The small diaphragm is then put in its place, and the flat centered by means of the adjustment of the large mirror, or, better still, by the adjustment of the eye end.

Thus the telescope can be centered in any position with accuracy during the day or with illumination at night.

The sizes of the diaphragms are proportionate to the diameters of the large and small mirrors respectively, and the length of the tube is a little less in proportion than the focal length so as to embrace a rather larger angle.

At night time a reflector is necessary within the tube at an angle of 45° to the tube, with an opening at the side so as to throw the light of a lamp held in the hand upon the inner face of the diaphragm, and a white sheet of paper stretched on a hoop can be set in front of the telescope and illuminated by another lamp. This, however, is not necessary if the work is properly done in the daytime.

Mean Places of Comparison Stars.

Star's Name.	R.A. 1887 ^o .			N.P.D. 1887 ^o .			Authority.
	h	m	s	°	'	"	
(a) W.B. 01043	1	0	44 81	82	14	37.9	Glasgow Catalogue, 1875.
(b) Arg. Zone + 12°, No. 242.	1	44	56.64	77	39	4.1	Bonn Observations, vol. vi.
(c) Oeltz.-Arg. (N.) 3731	3	16	22.43	31	42	31.3	Oeltz.-Arg. (N.)
(d) Oeltz.-Arg. (N.) 3772-3	3	19	37.50	32	43	18.6	" "
(e) Anonymous							" "
(f) Arg. Zone + 46°, No. 3148	20	58	0	43	2		Bonn Observations, vol. v.
(g) Arg. Zone + 46°, No. 3148	20	58	0	43	2		" " vol. v.
(h) Arg. Zone + 22°, No. 767	4	45	56	67	3		" " vol. iv.

Notes.

Jan. 16 and 25. Comet very faint.

Feb. 28. Comet extremely faint, partly owing to moonlight.

The observations are corrected for parallax and refraction. The initials H. T., M., and H. are those of Mr. Turner, Mr. Maunder, and Mr. Hollis respectively.

Royal Observatory, Greenwich:
1887, March 11.

Observations of Comets made at the Royal Observatory, Cape of Good Hope, in the year 1886.

(Communicated by D. Gill, LL.D., F.R.S., H.M. Astronomer.)

All the observations were made with the Repsold filar micrometer of the 7-inch Equatorial, and they are corrected for the effects of differential refraction.

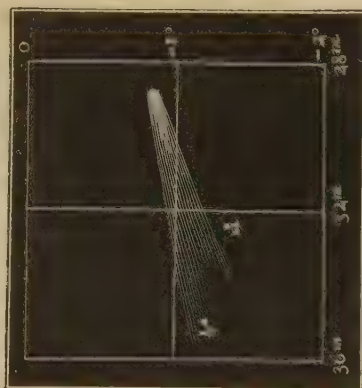
The observations have been chiefly made by Mr. W. H. Finlay : a few observations of Fabry's Comet by Dr. Gill are marked G.

Comet 1886 . . . (Fabry).

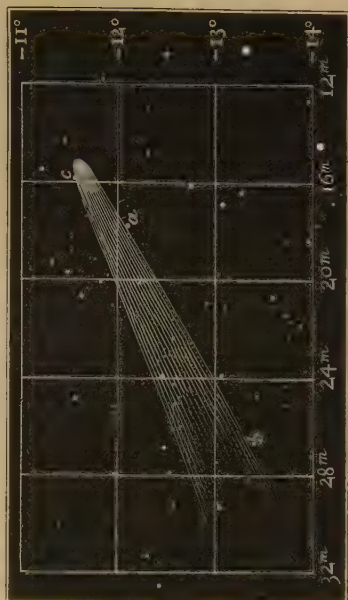
1886.	Cape Mean Time.			Comet—Star		No. of Comps.	Obser- ver.	Comet's App. R.A.			Log. ($p \times \Delta$)	Comet's App. Dec.		Log. ($p \times \Delta$)	Rel. to App. Place.	Comp. Star No.
	h	m	s	$\Delta \alpha$	$\Delta \delta$			m	s	"		$^{\circ}$	'	"		
May 1	6	39	12.3	+1	20.42	+6	3.0	4	6	36.01	9.6821	+5	19	22.2	-0.56	-11.3 1
2	6	50	24.5	-0	40.19	-0	42.0	4	32	9.58	9.6757	-0	38	26.0	-0.55	-12.8 2
	7	6	57.9	-0	23.52	-4	38.5	4	32	26.25	9.6826	-0	42	22.5	-0.55	-12.8 2
3	6	52	17.2	+0	59.52	+2	6.4	4	56	8.07	9.6671	-6	13	26.9	-0.55	-14.2 3
4	6	33	32.8	-1	18.73	-4	15.5	5	18	0.42	9.6424	-11	10	15.6	-0.53	-15.8 4
	7	22	21.2	+0	31.78	-0	51.4	5	18	43.70	9.6805	-11	19	45.8	-0.54	-15.7 5
5	6	26	37.3	+1	40.57	-5	49.3	5	38		9.6236	-15	29		-0.52	-17.0 6
6	6	46	45.1	+0	40.54	+1	17.8	5	56	34.37	9.6412	-19	13	32.0	-0.49	-18.3 7
10	7	58	41.1	-1	58.12	+4	12.6	6	52	10.28	9.7055	-28	45	12.7	-0.32	-21.8 8
11	8	21	13.6	-0	50.74	+6	1.3	7	2	30.94	9.7246	-30	13	40.8	-0.30	-22.3 9
12	8	13	31.4	+1	53.34	+0	24.5	7	11	34.51	9.7199	-31	26	45.3	-0.28	-22.5 10
1	7	49	13.4	-1	4.58	-1	14.6	7	19	37.77	9.6959	-32	28	14.6	-0.23	-23.0 11

1886.	Cape Mean Time.	Comet—Star $\Delta\alpha$	No. of Comps.	Obser- ver.	Comet's App. R.A. h m s	Log. ($p \times \Delta$)	Comet's App. Dec.	Log. ($p \times \Delta$)	Red. to App. Place.	Comp. Star No.
May 16	h m s 7 57 54.4	m s -0 28.57	16.12	F	7 39 53.61	9.7090	-34 48 34.8	0.2723 _n	-0.16	23.6 12
17	7 46 35.6	+0 49.95	20.16	F	7 45 26.57	9.6963	-35 23 33.6	0.2063 _n	-0.14	23.7 13
21	7 59 48.0	+0 14.74	10.8	F	8 3 48.69	9.7184	-37 9 43.5	0.1981 _n	-0.08	23.8 14
22	7 43 14.5	-1 5.92	20.16	F	8 7 34.73	9.7001	-37 29 43.2	0.1080 _n	-0.05	24.1 15
28	8 7 31.9	-1 49.21	16.12	F	8 26	9.7436	-39	0.2067 _n	0.00	23.9 16
29	7 53 55.6	+0 18.28	20.16	F	8 28 53.45	9.7308	-39 13 35.1	0.1331 _n	-0.02	23.9 17
June 6	8 25 16.4	-1 5.08	12.12	F	8 46 37.21	9.7817	-40 32 24.9	0.3180 _n	-0.01	23.2 18
7	7 33 27.6	+0 40.39	12.12	F	8 48 29.15	9.7352	-40 40 40.6	0.0545 _n	-0.01	23.2 19
10	8 12 3.6	-0 13.48	12.12	F	8 54 3.37	9.7817	-41 5 28.7	0.2862 _n	-0.01	22.9 20
12	7 45 11.1	+0 24.82	12.12	F	8 57 31.58	9.7646	-41 21 12.0	0.0713 _n	-0.01	22.7 21
22	8 3 40.1	+0 5.51	20	F	9 13 35.71	9.8053			-0.02	21.6 22
	8.14 42.3	+0 3.2	6	F			-42 38 33.0	0.3854 _n	-0.02	21.6 22
23	8 11 2.5	-0 49.07	12	F	9 15 6.72	9.8109			-0.02	23
	8 11 2.5	-1 37.0	16	F			-42 46 16.1	0.3857 _n	-0.02	21.5 24
26	7 25 41.7	-2 21.1	6	F			-43 9 29.2	0.1927 _n	-0.03	21.1 25
	7 34 1.0	+0 25.46	8	F	9 19 32.82	9.7959			-0.03	21.1 25
July 3	7 29 49.3	+0 26.80	10.10	F	9 29 28.86	9.8117	-44 6 36.8	0.2844 _n	-0.04	20.1 26
5	8 4 9.9	-0 58.7	4	F			-44 23 13.7	0.4395 _n	-0.06	19.8 27
	8 13 20.4	+1 34.23	4	F	9 32 31.29	9.8337				27
6	7 4 3.0	-1 33.3	12.12	F	9 33 53.06	9.8019	-44 31 24.7	0.1791 _n	-0.04	19.7 28
7	7 3 30.5	-0 7.0	10	F			-44 39 55.3	0.1868 _n	-0.05	19.6 29

1886.	Cape Mean Time.	Comet—Star		No. of Comps.	Obser- ver.	Comet's App. R.A.		Log. ($p \times \Delta$)	Comet's App. Dec.	Log. ($p \times \Delta$)	Red. to App. Place.	Comp. Star No.
	$h\ m\ s$	$\Delta\alpha$	$\Delta\delta$			$h\ m\ s$	$^{\circ}$		$^{\circ}$		s	
July 7	7 26 57.8	+0 11.26	"	14	F	9 35 17.37		9.8200			"	29
8	7 19 20.8	-0 23.15	+0 47.0	10.12	F	9 36 41.40	-44 48 44.9	0.2800 _n	-0.04		-19.4	30
20	7 26 53.2	-0 18.26	-1 10.2	12.8	F	9 53 11.98	-46 39 33.1	0.4194 _n	-0.06		-17.6	31
26	7 40.10.0	-0 51.20	-0 17.8	8.8	F	10 1 23.37	-47 39 37.4	0.5069 _n	-0.06		-16.6	32
30	7 54 42.4	-0 14.50	+1 0.2	6.4	F	10 6 48.28	-48 21 29.6	0.5760 _n	-0.07		-15.9	33



Comet Fabry, May 2, 7^h 15^m Cape Meantime
(on Argelander's Map.)



Comet Fabry, May 4, 8^h 5^m Cape Meantime
(on the Berlin Star Maps).

Notes.

- May 4. Excellent observations: star-like nucleus of 9th magnitude: admits of as easy observation as a star.
 5. Observations difficult on account of haze.
 21. Haze constantly blotting out comet.
 July 5. Comet near a 10½ magnitude star and very faint. Definition exceedingly bad.
 6, 8. Very faint: moonlight.
 12, 13. Good observations: star-like nucleus of 9½ magnitude.
 June 22. Comet faint.
 26, 30. Small and faint.

Comet 1886 . . . (Fabry).

Adopted Mean Places of Comparison Stars.

Comp. Star No.	R.A. 1886'o.	Declination 1886'o.	Authority.
	^h ^m ^s	[°] ['] ["]	
1	4 5 16.15	+ 5 13 30.5	Greenwich 7-year and 9-year.
2	4 32 50.32	- 0 37 31.2	Göttingen, 1263.
3	4 55 9.10	- 6 15 19.1	8½ mag. Equat. diff. from <i>a</i> .
"	4 54 45.61	- 6 21 44.3	W.B. iv. 1177.
4	5 19 19.68	- 11 5 44.3	½ (W.B.V. 407 + Sant. 519).
5	5 18 12.46	- 11 18 38.7	½ (W.B.V. 382 + Sant. 516).
6	5 36 (27)	- 15 23 (42)	(= Lal. 10807).
7	5 55 54.32	- 19 14 31.5	9½ mag. Equat. diff. from <i>b</i> .
<i>b</i>	6 2 43.92	- 19 9 13.5	Stone 2801.
8	6 54 8.72	- 28 49 3.5	ε Can. Maj. N.A.
9	7 3 21.98	- 30 19 19.8	Cord. Z. vii. 209.
10	7 9 41.45	- 31 26 47.3	Cord. Z. vii. 626.
11	7 20 42.58	- 32 26 37.0	8½ mag. Equat. diff. from <i>c</i> .
<i>c</i>	7 17 49.07	- 32 22 13.3	Stone 3602.
12	7 40 22.34	- 34 51 25.3	Cord. Z. vii. 2889.
13	7 44 36.76	- 35 27 14.4	8 mag. Equat. diff. from <i>d</i> .
<i>d</i>	7 46 7.81	- 35 30 4.8	Cord. Z. vii. 3384.
14	8 3 34.03	- 37 5 42.3	Cord. Z. viii. 242.
15	8 8 40.70	- 37 34 53.1	Stone 4207.
16	8 28	- 38 57	9 mag.
17	8 28 35.19	- 39 11 56.5	Cord. Z. viii. 2355.
18	8 47 42.30	- 40 33 28.8	Stone 4707.
19	8 47 48.77	- 40 38 12.2	C. Z. viii. 3879 and Eq. diff. from 18.
20	8 54 16.86	- 41 0 6.1	10 mag. Equat. diff. from <i>e</i> .
<i>e</i>	8 53 20.89	- 40 54 36.2	Cord. Z. viii. 4288.
21	8 57 6.77	- 41 25 1.6	Stone 4803.
22	9 13 30.22	- 42 38 14.6	10 mag. Equat. diff. from <i>f</i> .
<i>f</i>	9 16 29.32	- 42 35 56.1	Cord. Z. ix. 1284.
23	9 15 55.81	- 42 48 49	dupl. 10 mag. Eq. diff. from 24.
24	9 15 8.43	- 42 44 17.6	Cord. Z. ix. 1175.
25	9 19 7.39	- 43 6 47.0	Cord. Z. ix. 1498.
26	9 29 2.10	- 44 3 36.5	10 mag. Equat. diff. from <i>g</i> .
<i>g</i>	9 25 1.44	- 44 2 25.4	Cord. Z. ix. 1985.
27	9 30 57.12	- 44 21 55.2	9 mag. Equat. diff. from <i>h</i> .
<i>h</i>	9 30 22.77	- 44 18 28.9	Cord. Z. ix. 2418.
28	9 34 19.62	- 44 29 31.7	9½ mag. diff. from C.Z. ix. 2776.
29	9 35 6.16	- 44 39 28.7	Cord. Z. ix. 2746.
30	9 37 4.59	- 44 49 12.5	10½ m. diff. from C.Z. ix. 2715.
31	9 53 30.30	- 46 38 5.3	10½ m. diff. from C.Z. ix. 4184.
32	10 2 14.63	- 47 39 3.0	10½ m. diff. from C.Z. x. 182.
33	10 7 2.85	- 48 22 13.9	Cord. Z. x. 493.

Observations of Comet 1886 . . . (Barnard).

1886.	Cape Mean Time.	Comet—Star.	No. of Comps.	Obser- ver.	Comet's App. R.A.	Log. ($p \times \Delta$)	Comet's App. Dec.	Log. ($p \times \Delta$)	Red. to App. Place.	Comp. Star No.
	h m s	$\Delta \alpha$			h m s					
May 29	6 1 20.9	+0 47.59	8.8	F	4 34 29.41	9.7095	-16 57 50.5	0.6685 _n	-0.71	9.2 1
June 1	6 10 6.7	-1 25.32	6.4	F	5 12 2.46	9.7413	-27 42 52.4	0.6059 _n	-0.88	11.1 2
5	6 37 12.8	+0 28.98	4.4	F	5 59 37.69	9.7890	-37 40 51.2	0.5288 _n	-1.04	13.8 3
6	6 22 18.8	-0 11.20	20.16	F	6 10 23.00	9.7942	-39 27 34.5	0.4596 _n	-1.06	14.5 4
7	6 8 34.6	-6 16.2	6	F			-41 1 2.4	0.3609 _n	-1.06	15.2 5
	6 23 14.6	-1 19.88	10	F	6 20 46.59	9.8008				5 6
	6 35 45.2	-0 10.94	10	F	6 20 52.07	9.8058			-1.06	15.4 6
	6 44 44.5		6	F			-41 3 14.7	0.4818 _n		6 6
10	6 42 22.7	+1 29.90	12.8	F		9.8274		0.3716 _n	-1.08	16.7 7
	7 13 50.4	-0 17.07	10.8	F		9.8366		0.4833 _n	-1.07	16.8 8
12	6 55 56.0	-0 40.77	20.12	F	7 6 10.	9.8433	-46 27 34.5	0.3656 _n	-1.05	17.7 9
15	7 31 39.5	-2 47.7	.4	F			-48 24 58.9	0.4405 _n	-1.01	18.5 10
	7 41 38.4	+1 31.32	6.	F	7 28 32.35	9.8677			-1.01	18.5 10
22	8 42 40.7	+0 42.2	.4	F				0.6026 _n	-0.81	20.1 11
	8 54 54.4	-4 5.35	4.	F		9.8821				11 11
23	7 26 39.9	+0 11.38	14.12	F	8 13 50.68	9.8880	-51 20 42.6	0.3162 _n	-0.82	19.9 12
July 5	7 25 17.0	+1 49.85	10.8	F	8 59 46.66	9.9090	-53 28 39.9	0.2781 _n	-0.64	19.8 13

1886.	Cape Mean Time.	Comet—Star $\Delta\alpha$	No. of Comps.	Obscr- ver.	Comet's App. R.A.	Log. ($\mu \times \Delta$)	Comet's App. Dec.	Log. ($\rho \times \Delta$)	Red. to App. Place.	Comp. Star No.
		$\Delta\delta$			$^h\ m\ s$		$^h\ m\ s$		^{s}	
July 6	7 58 33.9	-0 7.63	19.12	F	9 2 58.08	9.9169	53 36 59.8	0.4407 _n	-0.60	19.8 14
7	8 0 27.6	+1 23.23	10.14	F	9 6 0.49	9.9184	53 44 59.5	0.4514 _n	-0.61	19.6 15
8	7 58 4.9	+0 9.01	9.8	F	9 8 59.21	9.9197	53 52 50.9	0.4459 _n	-0.58	19.6 16
20	8 2 45.8	+5 52.1	4	F			55 24 8.6	0.5103 _n	-0.46	18.4 17
	8 18 21.2	-1 2.46	9	F	9 40 9.90	9.9317				17
21	6 34 48.6	-1 23.3	4	F			55 31 24.0	9.9816 _n	-0.46	18.4 17
	6 41 27.1	+1 7.52	4	F	9 42 19.88	9.9207				17
24	7 25 50.1	-3 4.2	4	F			55 55 2.1	0.3768 _n	-0.44	17.8 18
	7 40 6.5	-0 10.49	12	F	9 49 14.04	9.9417				18
26	8 20 41.0	-1 8.03	6.6	F	9 53 44.16	9.9363	56 11 1.7	0.5938 _n	-0.42	17.6 19

Notes.

June 15. Moonlight too strong.

July 6. Faint; moonlight

20. Very faint, and so mixed up with two stars that it was almost impossible to fix its position.

21. Cloudy night: observations not of much value.

26. An exceedingly faint and ill-defined patch of light.

July 5. Faint: very bad definition.

8. Large, faint and diffused: difficult.

24. Exceedingly faint: definition very bad.

Comet 1886 . . . (Barnard I.)

Adopted Mean Places of Comparison Stars.

Comp. Star No.	R.A. 1886.			Declination 1886.			Authority.
	h	m	s	°	'	"	
1	4	33	42.53	-16	57	14.0	Oeltz, Arg. S. 3251.
2	5	13	28.66	-27	36	26.6	Oeltz, Arg. S. 3846.
3	5	59	9.75	-37	32	40.1	Stone 2762.
4	6	10	35.26	-39	24	59.2	9½ mag. = Stone 2926 { -5 ^m 27 ^s .23. +1' 12''.2.
5	6	22	7.53	-40	54	31.0	Stone 2995.
6	6	21	4.07	-41	5	39.0	10 mag. = Stone 3017 { -3 ^m 5 ^s .00. -1' 51''.3.
7	6	48	(6)	-44	38		9 mag.
8	6	49	(53)	-44	46		9 mag.
9	7	6	52.	-46	23	55.4	10 mag. = C.Z. vii. 260 { +2 ^m 53 ^s .28. +2' 17''.4.
10	7	27	2.04	-48	21	52.7	Cord. Z. vii. 1876.
11	8	13	(9)	-51	6		9 mag.
12	8	14	2.88	-51	20	44.9	8½ mag. = C.Z. viii. 1471 { -3 ^m 57 ^s .87. -0' 44''.6.
13	8	57	57.45	-53	33	40.5	10 mag. = C.Z. viii. 4702 { -0 ^m 22 ^s .17. +5' 42''.7.
14	9	3	6.31	-53	37	53.4	C.Z. ix. 229.
15	9	4	37.87	-53	49	54.3	8½ mag. = C.Z. ix. 416 { -0 ^m 36 ^s .60. +4' 36''.2.
16	9	8	50.78	-53	48	37.6	C.Z. ix. 666.
17	9	41	12.82	-55	29	42.3	C.Z. ix. 3183.
18	9	49	24.97	-55	51	40.1	C.Z. ix. 3790.
19	9	54	52.61	-56	7	42.3	C.Z. ix. 4203.

Comet Brooks I. (1886).

1886.	Cape Mean Time.		Comet-Star.		No. of Comps.	Obs. ver.	Comet's App. R.A.		Log. ($p \times \Delta$)	Comet's App. Decl.		Log. ($p \times \Delta$)	Red. to App. Place.	Comp. Star No.
	h	m	s	$\Delta \delta$			h	m		o	'		s	
July 5	6	31	14.3	-1 22'96	-1 30'5	6.4	F	8 30'41.86	9.6810	9 15	26.1	0.6589 _n	-0.03	1
6	6	18	46.5	-0 27'10	+3 53.1	16.16	F	8 36'56.66	9.6735	-	9 29	57.7	-0.01	2
7	6	26	17.1	+0 31'60	-2 36.	20.16	F		9.6768	-	9 29	57.7	0.00	3
8	6	33	31.7	+1 3'55	+1 17.6	16.12	F		9.6799	-	9 29	57.7	+0.02	4
9	6	30	55.9	+0 1'33	+0 27.2	30.16	F		9.6774	-	9 29	57.7	+0.04	5
12	6	48	53.1	-2.29'03	-7 29.3	4.4	F	9 5 15.51	9.6853	-10 27	24.8	0.6552 _n	+0.10	6
14	6	32	30.5	+1 3'07	-2 6.4	10.8	F	9 15 17.89	9.6755	-10 44	58.6	0.6432 _n	+0.13	7
	6	32	30.5	-0 46'02	+0 7.7	10.8	F		9.6755	-	10 44	58.6	+0.13	8
20	6	39	1.1	+0 54'38	-1 32.0	16.10	F	9 42 9.29	9.6789	-11 26	23.8	0.6417 _n	+0.22	9
	6	39	1.1	-0 38'05	+1 14.2	16.10	F	9 42 9.21	9.6789	-11 26	29.0	0.6417 _n	+0.22	10
24	6	49	10.1	+0 33'75	-2 5.2	14.8	F	9 57 40.53	9.6849	-11 47	27.4	0.6461 _n	+0.27	11
26	6	41	26.3	+1 45'79	+1 16.3	10.8	F	10 4 47.09	9.6815	-11 56	33.8	0.6409 _n	+0.29	12
28	6	37	15.7	-0 52'36	+1 4.0	8.6	F	10 11 33.04	9.6800	-12 5	7.5	0.6382 _n	+0.32	13
29	6	41	55.1	+0 19'19	-1 23.9	10.8	F	10 14 50.26	9.6829	-12 9	8.9	0.6411 _n	+0.33	14
30	6	46	25.4	-1 49'76	-1 57.1	8.8	F	10 18 2.49	9.6861	-12 13	12.5	0.6446 _n	+0.35	15

Notes.

July 5. A bright circular mass, about $\frac{3}{4}$ in diameter, gradually condensed toward the centre.
 12. Hazy sky: observations rough.

20. A diffused mass without any particular condensation.

24. Definition bad. A rather faint diffused patch of light: does not admit of very accurate observation.

29. Faint and ill-defined. A difficult object to observe.

14. Comet very faint: bright moonlight.

30. Very faint.

Comet 1886 . . . (Brooks I.).

Adopted Mean Places of Comparison Stars.

Comp. Star No.	R.A. 1886 ^o .			Declination 1886 ^o .			Authority.
	^h	^m	^s	^o	[']	["]	
1	8	25	(42)	— 9	3	"	8 mag.
2	8	31	8.97	— 9	19	7.6	Yarnall 3507.
3	8	36	25.06	— 9	27	42.6	9 mag. Equat. diff. from <i>a</i> .
<i>a</i>	8	37	23.63	— 9	23	49.5	W.B. viii. 942.
4	8	41	(48)	— 9	44		10½ mag. = <i>b</i> { + 1 ^m 27 ^s .40.
<i>b</i>	8	40	(21)	— 9	49		+ 5' 20 ^{''} .9.
5	8	49	(7)	— 9	57		10½ mag. = <i>c</i> { — 2 ^m 25 ^s .30.
<i>c</i>	8	51	(32)	— 9	55		— 1' 49 ^{''} .4.
6	9	7	44.44	— 10	19	44.1	9 mag.
7	9	14	14.69	— 10	42	41.0	Santini 1142.
8	9	16	(3)	— 10	45		Santini 1155.
9	9	41	14.69	— 11	24	41.1	9 mag.
10	9	42	47.04	— 11	27	32.5	½ (Yarnall + Santini).
11	9	57	6.51	— 11	45	11.9	Santini 1199.
<i>d</i>	10	0	40.92	— 11	51	37.6	10½ mag. diff. from <i>d</i> .
<i>e</i>	10	1	6.25	— 11	55	5.1	9½ mag. diff. from <i>e</i> .
12	10	3	1.01	— 11	55	5.1	W.B. ix. 1277.
13	10	12	25.08	— 11	57	40.0	½ (Schj. + diff. from λ Hydræ).
14	10	14	30.74	— 12	6	1.7	9½ mag. diff. from W.B.X. 114 and X. 204.
15	10	19	51.90	— 12	7	35.3	10 mag. diff. from * 13 and Lam. 711.
				— 12	11	5.8	Lamont Z. 711.

Winnecke's Comet.

1886.	Cape Mean Time.	Comet - Star. $\Delta\alpha$	No. of Comps.	Obser- ver.	Comet's App. R.A.	Log. ($p \times \Delta$)	Comet's App. Decl.	Log. ($p \times \Delta$)	Red. to App. Place.	Comp. Star No.
	^{h} ^{m} ^{s}	^{s} $^'$ $''$			^{h} ^{m} ^{s}		$^{\circ}$ $'$ $''$		^{s}	
Aug. 19	8 12 28.8	-0 44.77	3.5	F	13 7 6.07	9.674	-0 34 2.8	0.690 _n	+0.83	1
20	7 1 49.9	+0 50.87	16.12	F	13 10 21.47	9.613	-1 8 17.8	0.684 _n	+0.84	2
20	7 39 2.0	-1 10.10	10.8	F	13 10 26.82	9.651	-1 9 14.0	0.686 _n	+0.85	3
21	7 12 7.4	+1 37.91	12.12	F	13 13 50.77	9.626	-1 45 1.9	0.681 _n	+0.85	4
22	6 47 5.3	+0 13.99	12.12	F	13 17	9.595	-2 21	0.675 _n	+0.87	5
25	7 35 35.4	-1 18.82	11.8	F	13 28 11.92	9.651	-4 16 3.0	0.670 _n	+0.92	6
29	7 28 34.7	+0 27.69	12.12	F	13 43 9.99	9.648	-6 53 12.4	0.653 _n	+0.98	7
Sept. 4	7 34 15.1	-0 35.33	12.12	F	14 7 9.98	9.658	-11 1 3.0	0.627 _n	+1.02	8
16	7 43 5.3	+5 6.4	0.4	F			-19 41 47.4	0.558 _n	+1.27	9
16	7 49 59.2	+1 27.7	3.0	F	15 1 51.61	9.682			+1.27	9
17	7 38 19.5	-1 42.06	16.12	F	15 6 50.35	9.674	-20 25 11.6	0.544 _n	+1.31	10
18	7 36 44.7	+0 46.16	16.12	F	15 11 56.35	9.673	-21 8 34.7	0.533 _n	+1.31	11
19	7 31 29.1	-0 9.45	8.8	F	15 17 7.49	9.669	-21 51 35.9	0.516 _n	+1.34	12
20	7 25 58.0	-0 1.36	26.12	F	15 22 22.77	9.664	-22 34 19.7	0.498 _n	+1.36	13
25	7 52 47.7	-1 50.32	16.12	F	15 50 12.10	9.695	-26 3 16.4	0.485 _n	+1.47	14
26	7 40 27.8	-0 51.32	16.12	F	15 55 58.06	9.684	-26 42 54.9	0.450 _n	+1.49	15
27	9 9 22.4	+1 5.14	10.8	F	16 2 14.06	9.739	-27 24 32.6	0.598 _n	+1.51	16
30	8 50 22.4	+0 12.83	16.12	F	16 20 24.50	9.738	-29 16 36.0	0.541 _n	+1.57	17
Oct. 1	8 32 3.4	-3 19.3	0.4	F			-29 51 48.8	0.494 _n	+1.59	18
1	8 41 7.9	+1 43.80	3.0	F	16 26 37.56	9.735			+1.59	18
15	8 23 40.9	-0 39.12	12.12	F	18 1 2.38	9.718	-35 54 33.6	0.257 _n	+1.88	19

1886.	Cape Mean Time.	Comet—Star $\Delta\alpha$	No. of Comps.	Obs- vet.	Comet's App. R.A.	Log. ($p \times \Delta$)	Comet's App. Dec.	Log. ($p \times \Delta$)	Red. to App. Place.	Comp. Star No.
	h m s	' "			h m s		o i "		s "	
Oct. 18	8 28 25.2	+1 24.7	16.12	F	18 29 8.26	9.717	-36 42 44.7	0.225 _n	+1.91	+ 4.5 20
19	8 39 2.7	+4 10.5	16.12	F	18 29 8.26	9.727	-36 42 44.7	0.253 _n	+1.94	+ 4.8 21
21	9 8 57.4	-0 37.7	16.12	F	18 43 10.21	9.751	-36 57 8.9	0.337 _n	+1.95	+ 5.2 22
22	8 14 23.9	-0 46.1	12.12	F	18 49 48.25	9.686	-37 1 48.8	0.084 _n	+1.96	+ 5.4 23
29	9 1 40.3	-0 43.0	0.4	F	19 36 37		-36 54 20.0	0.220 _n	+2.01	+ 6.7 24
31	8 30 21.6	+0 5.7	0.4	F				0.049 _n	+2.03	+ 7.1 25
31	8 41 4.9	-0 18.65	6.0	F		9.688		+2.03	+ 7.1	25
Nov. 2	8 12 45.7	-1 11.85	10.8	F	20 1 13.60	9.633	-36 22 14.9	9.938 _n	+2.03	+ 7.5 26
13	8 1 12.3	+2 37.9	0.4	F				9.935 _n	+2.04	+ 9.0 27
13	8 11 32.1		12.0	F		9.581		+2.04	+ 9.0	27
17	9 15 15.6		0.4	F				0.283 _n	+2.04	+ 9.5 28
17	9 23 36.6	+2 36.4	10.0	F	21 10 35.42	9.690	-32 23 3.5	+2.04	+ 9.5	28
19	9 42 58.4	-0 7.8	5.5	F	21 29 33.74	9.707	-31 42 30.0	0.375 _n	+2.04	+ 9.5 28
23	8 37 8.6	-0 51.7	8.8	F	21 46 9.33	9.607	-30 21 47.3	0.212 _n	+2.03	+ 9.6 29
25	8 55 16.9	-1 8.1	8.10	F	21 54 7.76	9.635	-29 36 59.8	0.287 _n	+2.05	+10.0 30
26	9 21 36.0	+0 10.3	10.8	F	21 58 3.11	9.671	-29 15 12.2	0.363 _n	+2.03	+10.1 31
29	9 9 15.4	+5 39.9	10.8	F	22 9 7.47	9.651	-28 10 23.9	0.357 _n	+2.04	+10.1 32
								+2.03	+10.1	33

Notes.

Aug. 25. Observations made through cloud: not very good.

Sept. 16. Comet seen during a short break in the clouds.

Oct. 1. Moonlight and clouds; observations not very valuable.

Nov. 2. Faint in the moonlight.

Sept. 4. Bright moonlight: comet faint.

18. Well condensed towards centre, but scarcely a stellar nucleus.

Oct. 29. Clouds prevented further observations.

Nov. 17. Faint from this date onwards.

Winnecke.

Adopted Mean Places of Comparison Stars.

Comp. Star No.	R.A. 1886°.	Declination 1886°.	Authority.
	h m s	° ' "	
1	13 7 50.01	— 0 30 56.2	10 mag. Equat. diff. from * a.
a	13 13 7.91	— 0 40 3.1	Copeland and Börgen 3986.
2	13 9 29.76	— 1 8 33.9	Copeland and Börgen 3979.
3	13 11 36.07	— 1 4 56.8	Copeland and Börgen 3982.
4	13 12 12.01	— 1 44 34.8	9 mag. one Equat. diff. from * b.
b	13 18 31.41	— 1 42 54.3	Copeland and Börgen 3996.
5	13 16 (58)	— 2 20	10 mag.
6	13 29 29.82	— 4 20 58.5	W.B. xiii. 465.
7	13 42 41.32	— 6 56 12.3	10½ mag. Equat. diff. from * c.
c	13 44 33.32	— 7 1 50.7	Schjel. 4937-8.
8	14 7 44.29	— 11 5 33.5	10 mag. Equat. diff. from * d.
d	14 4 34.19	— 11 5 55.9	W.B. xiv. 33.
9	15 0 47.57	— 19 46 54.0	Oeltz. Arg. S. 14264-5.
10	15 8 31.10	— 20 24 55.9	9½ mag. Equat. diff. from * e.
e	15 9 13.15	— 20 30 50.7	Oeltz. Arg. S. 14402.
11	15 11 8.88	— 21 9 7.6	Wash. Zones 255-18.
12	15 17 15.60	— 21 56 40.7	9 mag. Equat. diff. from * f.
f	15 14 56.45	— 21 55 55.1	Oeltz. Arg. S. 14484.
13	15 22 22.77	— 22 32 4.8	9½ mag. Equat. diff. from * g.
g	15 20 30.67	— 22 29 23.1	Oeltz. Arg. S. 14561.
14	15 52 0.95	— 26 0 59.3	9 mag. Equat. diff. from * h.
h	15 48 36.85	— 25 55 43.6	Stone 8647.
15	15 56 47.89	— 26 40 41.1	½ (C.Z. 3944 + Eq. diff. C.Z. 4023).
16	16 1 7.41	— 27 25 27.3	C.Z. xvi. 30.
17	16 20 10.10	— 29 12 14.0	½ (C.Z. 1357 + Eq. diff. C.Z. 1559).
18	16 24 52.17	— 29 48 31.1	½ (C.Z. 1653 + Eq. diff. C.Z. 1592).
19	18 1 39.62	— 35 56 22.6	9½ mag. Equat. diff. from * k.
k	17 58 41.47	— 36 1 36.7	½ (Stone 9855 + C.Z. 3937).

Comp. Star No.	R.A. 1886°.	Declination 1886°.	Authority.
	h m s	° ' "	
20	18 22 (13)	-36 35	$9\frac{1}{2}$ mag. = * <i>l</i> $\left\{ \begin{array}{l} + 2^m 16^s.00. \\ - 1' 28''.7. \end{array} \right.$
<i>l</i>	18 19 (57)	-36 33	9 mag.
21	18 29 35.34	-36 47 0.0	$9\frac{1}{2}$ mag. Equat. diff. from * <i>m</i> .
<i>m</i>	18 36 25.26	-36 49 39.5	Stone 10182.
22	18 42 48.08	-36 56 36.4	C.Z. xviii. 2375.
23	18 48 27.34	-37 1 8.1	C.Z. xviii. 2633.
24	19 34 39.04	-36 53 43.7	$\frac{1}{2}$ (C.Z. 1443 + Stone 10615).
25	19 49 (28)	-36 40	$9\frac{1}{2}$ mag.
26	20 2 23.42	-36 21 57.0	$9\frac{1}{2}$ mag. Equat. diff. from * <i>n</i> .
<i>n</i>	20 3 42.19	-36 23 12.4	Stone 10813.
27	21 2 (0)	-33 45	$9\frac{1}{2}$ mag. = * <i>o</i> $\left\{ \begin{array}{l} - 0^m 37^s.50. \\ - 4' 42''.2. \end{array} \right.$
<i>o</i>	21 2 (38)	-33 40	$9\frac{1}{4}$ mag.
28	21 10 10.21	-32 25 49.4	C.Z. xxi. 608.
29	21 27 45.01	-31 42 31.8	$9\frac{1}{2}$ mag. one Equat. diff. from * <i>p</i> .
<i>p</i>	21 35 15.81	-31 46 43.8	C.Z. xxi. 1065.
30	21 45 34.40	-30 21 5.6	10 mag. Equat. diff. from * <i>q</i> .
<i>q</i>	21 43 59.27	-30 16 40.5	C.Z. xxi. 1353.
31	21 52 56.62	-29 36 1.8	$\frac{1}{2}$ (C.Z. 1644 + Stone 11567).
32	21 59 42.80	-29 15 32.6	C.Z. xxi. 1870.
33	22 8 16.71	-28 16 13.9	$\frac{1}{2}$ (C.Z. 232 + Eq. diff. Stone 11657).

Comet 1886 e . . . (Finlay).

1886.	Unpo Mean Time.	Comet—Star. $\Delta\alpha$	No. of Comps.	Obs. ver.	Comet's App. R.A.	Log. ($p \times \Delta$)	Comet's App. Decl.	Log. ($p \times \Delta$)	Red. to App. Place.	Star of Comp.
	h m s	m s			h m s				"	
Sept. 26	9 14 40.8	-1 29.21	14.12	F	17 2 1.78	9.7046	-26 4 10.5	0.5069 _n	+1.82	+ 4.2 1
27	8 28 10.1	+0 49.66	12.10	F	17 4 20.63	9.6628	-26 7 8.4	0.4266 _n	+1.80	+ 4.3 1
27	9 38 24.5	+0 56.56	8.8	F	17 4 27.53	9.7220	-26 7 15.0	0.5528 _n	+1.80	+ 4.3 1
29	9 19 7.7	+1 55.16	6.6	F	17 9 19.54	9.7129	-26 13 4.9	0.5247 _n	+1.78	+ 4.5 2
30	7 56 12.6	+0 25.06	24.16	F	17 11 42.50	9.6253	-26 15 46.0	0.3690 _n	+1.78	+ 4.7 3
Oct. 1	7 53 16.0	-0 5.93	6.6	F	17 14 15.87	9.6231	-26 18 30.4	0.3647 _n	+1.78	+ 4.8 4
2	7 30 48.6	+1 54.2	0.4	F			-26 21 5.1	0.3212 _n	+1.76	+ 4.8 5
2	8 2 19.4	+2 31.03	15.0	F	17 16 53.96	9.6389			+1.76	+ 4.8 5
5	7 57 55.6	+2 3.21	10.8	F	17 24 59.05	9.6383	-26 28 1.4	0.3816 _n	+1.75	+ 5.2 6
10	7 43 26.3	+0 46.50	4.4	F	17 39 22.17	9.6246	-26 36 19.3	0.3597 _n	+1.74	+ 6.0 7
15	7 39 44.4	+0 13.67	16.12	F		9.6260		0.3594 _n	+1.72	+ 6.7 8
18	7 42 50.8	+0 36.83	20.12	F	18 4 48.96	9.6337	-26 38 6.7	0.3711 _n	+1.71	+ 7.2 9
19	7 42 36.9	-0 57.66	16.12	F		9.6339		0.3715 _n	+1.72	+ 7.4 10
21	8 0 49.3	-1 54.99	16.12	F	18 15 10.78	9.6600	-26 34 12.9	0.4119 _n	+1.72	+ 7.7 11
21	8 0 49.3	-2 24.65	16.12	F	18 15 10.78	9.6600	-26 34 12.0	0.4119 _n	+1.72	+ 7.7 12
22	8 15 25.9	+1 9.33	23.12	F	18 18 44.74	9.6774	-26 32 10.4	0.4422 _n	+1.70	+ 7.8 12
26	7 52 23.1	-1 28.60	10.8	F	18 33 17.51	9.6504	-26 20 39.8	0.4022 _n	+1.71	+ 8.4 13
29	7 46 36.0		0.4	F			-26 7 46.9	0.3958 _n	+1.70	+ 8.8 14
29	8 3 15.6	+0 34.82	10.0	F	18 44 44.74	9.6645			+1.70	+ 8.8 14
30	7 42 52.9		0.6	F			-26 2 40.7	0.3885	+1.70	+ 9.0 15
30	8 3 19.9	-0 7.99	14.0	F	18 48 38.33	9.6634			+1.70	+ 9.0 15
Nov. 13	8 49 3.9	-0 9.42	12.8	F	19 47 59.29	9.6967	-23 55 0.5	0.5288 _n	+1.71	+ 11.2 16

March 1887.

Observations of Comets.

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1886.	Cape		Comet-Star.		No. of Comps.	Obs- ver.	Comet's			Log. ($p \times \Delta$)	Comet's App. Dec.	Log. ($p \times \Delta$)	Red. to App. Place.	Star of Comp.
	h	m	s	Δ	Δ	δ	h	m	s					
Nov. 17	8 25	22.9	+0 50.63	-0 10.5	11.8	F	20	6	16.29	9.6701	-22 56 54.4	0.4994 _n	+1.70	17
19	8 8	4.2	+0 32.73	-2 19.5	16.12	F	20	15	36.22	9.6471	-22 23 45.8	0.4787 _n	+1.72	18
22	8 14	26.6	-0 45.30	+1 3.1	24.12	F	20	29	57.16	9.6490	-21 28 47.5	0.4965 _n	+1.74	19
23	8 7	41.4	-0 11.38	+3 38.9	20.12	F	20	34	46.43	9.6386	-21 9 13.2	0.4905 _n	+1.74	20
25	8 9	12.6	-0 28.86	-2 12.4	16.12	F	20	44	33.66	9.6360	-20 27 14.1	0.4996 _n	+1.76	21
26	8 8	58.8	-0 20.75	-4 40.3	16.12	F	20	49	28.77	9.6335	-20 5 15.9	0.5030 _n	+1.74	22
29	8 12	24.6	+0 26.75	+0 19.7	16.12	F	21	4	24.98	9.6308	-18 54 39.2	0.5182 _n	+1.78	23
Dec. 4	8 30	36.8		+3 34.0	0.8	F					-16 42 39.4	0.5569 _n	+1.81	24
6	8 50	4.2	+0 7.71	+1 18.1	20.12	F	21	40		9.6568	-15 47	0.5821 _n	+1.84	25
13	8 34	48.4	-1 31.33	+0 48.2	12.8	F	22	16	7.41	9.6235	-12 3 42.7	0.5987 _n	+1.92	26
15	8 36	46.0	-1 18.35	+0 38.1	22.12	F	22	26	27.86	9.6217	-10 55 42.1	0.6091 _n	+1.97	27
16	8 27	23.6	-0 25.81	+3 59.0	12.8	F	22	31	36.10	9.6067	-10 21 13.5	0.6086 _n	+1.98	28
17	8 30	20.5	-0 16.05	-1 24.1	18.12	F	22	36	46.97	9.6086	-9 46 7.6	0.6150 _n	+2.00	29
18	8 34	51.6	-0 26.72	+0 19.4	18.12	F	22	41	56.86	9.6121	-9 10 38.5	0.6219 _n	+2.01	30
23	8 46	45.6	-1 53.18	+0 6.3	14.12	F	23	7	38.95	9.6163	-6 9 9.3	0.6490 _n	+2.11	31
27	8 50	22.1	+0 50.44	-2 45.6	16.12	F	23	27	56.42	9.6138	-3 41 14.2	0.6668 _n	+2.18	32
28	8 47	1.9	-0 27.83	-0 24.6	16.12	F	23	32	(56)	9.6080	-3 5	0.6706 _n	+2.21	33

Notes.

Sept. 26. Faint, round, very slightly more condensed towards the centre.

27. The second set of observations is the better; during the first set the comet was approaching a star, 11 mag, which completely overpowered it at last.

Oct. 1. Hazy night: comet close enough to the comparison star to be rendered rather difficult.

5, 10. Moonlight

Oct. 26. Hazy.

Nov. 25. Strong S.E. wind: the central condensation seemed somewhat elongated in position-angle $165^\circ \pm$, but the definition was very bad this night.

29. Very bad definition: strong wind.

Dec. 4. Chronograph failed: signals not recorded.

Comet 1886 e . . . (Finlay).

Adopted Mean Places of Comparison Stars.

Comp. Star No.	R.A. 1886'o.			Declination 1886'o.			Authority.
	h	m	s	°	'	"	
1	17	3	29.17	-26	5	52.8	= A.O. 16394 comp. with C.Z. 373, 550.
2	17	7	22.60	-26	10	40.4	9½ mag. Equat. diff. from * 1.
3	17	11	15.66	-26	11	25.0	½ (C.Z. 728 + Equat. diff. from C.Z. 934).
4	17	14	20.02	-26	19	7.6	C.Z. xvii. 927.
5	17	14	21.17	-26	23	4.1	C.Z. xvii. 928.
6	17	22	54.09	-27	30	12.0	9½ mag. = C.Z. 1430 { + 1 ^m 10 ^s .36. + 7' 43''.4.
7	17	38	33.93	-26	36	0.0	9½ mag. = A.O. 17142 { + 0 ^m 51 ^s .30. - 3' 52''.3.
8	17	54	(37)	-26	38		9½ mag. = * a { - 3 ^m 37 ^s .05. + 0' 46''.9.
a	17	58	(14)	-26	39		9 mag.
9	18	4	10.42	-26	39	56.0	Wash. Zones 100-56 and 36-93.
10	18	9	(10)	-26	37		10 mag. = * b { - 0 ^m 59 ^s .90. - 1' 9''.3.
b	18	10	(10)	-26	36		9 mag.
11	18	17	4.05	-26	30	16.8	½ (C.Z. 1037 + Yarnall).
12	18	17	33.71	-26	33	3.3	½ (C.Z. 1071 + Yarnall).
13	18	34	44.40	-26	24	3.1	9½ mag. = C.Z. 1732 { + 5 ^m 0 ^s .70. - 1' 15''.4.
14	18	44	8.22	-26	8	56.4	C.Z. xviii. 2438.
15	18	48	44.62	-26	4	44.7	Wash. Zones 127-24.
16	19	48	7.00	-23	56	51.1	9½ mag. = C.Z. xix. 2002 { - 0 ^m 36 ^s .90. + 3' 38''.6.
17	20	5	23.96	-22	56	55.6	9½ mag. = A.O. 20264 { + 3 ^m 19 ^s .75. + 4' 25''.2.
18	20	15	1.77	-22	21	38.4	9½ mag. = A.O. 20436 { - 0 ^m 23 ^s .60. - 2' 44''.6.
19	20	30	40.72	-21	30	3.2	9½ mag. = * c { - 1 ^m 7 ^s .75. - 6' 41''.1.
c	20	31	48.47	-21	23	22.1	½ (A.O. 20678 + Wash. 1860).
20	20	34	56.07	-21	13	4.8	(= A.O. 20744) Wash Zones (2 obs.).
21	20	45	0.76	-20	25	14.7	10½ mag. Equat. diff. from * d.
d	20	40	6.16	-20	26	45.7	9¼ mag. Wash. Zones 190.
22	20	49	47.78	-20	0	48.6	9½ mag. Equat. diff. from * e.
e	20	45	44.88	-20	4	12.8	Radcliffe Obs. 1881.
23	21	3	56.45	-18	55	12.3	A.O. 21180.
24	21	28	55.51	-16	46	27.4	9 mag. Equat. diff. from * f.
f	21	27	22.21	-16	42	15.7	(= A.O. 21468) Edinburgh Catalogue.
25	21	39	(59)	-15	48		9½ mag. = * g { - 2 ^m 26 ^s .45. + 1' 31''.3.
g	21	42	(25)	-15	50		9 mag.
6	22	17	36.82	-12	4	45.8	10 mag. Equat. diff. from * h.
h	22	12	50.80	-12	7	38.7	½ (Schj. 9115 + Sant. 2507).
27	22	27	44.24	-10	56	35.2	9½ mag. Equat. diff. from * k.

Comp. Star No.	R.A. 1886°.			Declination 1886°.	Authority.
	^h	^m	^s		
<i>k</i>	22	31	13.30	— 10 51 39.1	$\frac{1}{2}$ (Yarnall 9932 + Sant. 2550).
28	22	31	59.93	— 10 25 27.6	$9\frac{1}{2}$ mag. 1 Equat. diff. from * <i>l</i> .
<i>l</i>	22	36	30.73	— 10 30 50.7	(= Lal. 44394-5) Paris Obs. 1862.
29	22	37	1.02	— 9 44 58.7	$\frac{1}{2}$ (Schj + Santini).
30	22	42	21.57	— 9 11 13.1	$9\frac{1}{2}$ mag. Equat. diff. from * <i>m</i> .
<i>m</i>	22	45	48.20	— 9 9 45.1	Schjellerup 9365.
31	23	9	30.02	— 6 9 31.0	10 mag. Equat. Diff. from * <i>n</i> .
<i>n</i>	23	12	2.17	— 6 8 22.5	9 mag. Astr. Nach. lxxvi. p. 215.
32	23	27	3.80	— 3 38 44.1	Karlsruhe Zones (Valentiner).
33	23	33	(22)	— 3 5	$9\frac{1}{2}$ mag.

Observations of Winnecke's Comet, 1886, made at Windsor, New South Wales. By John Tebbutt.

Notice of the discovery of this comet at the Cape of Good Hope was received here on August 25. The comet was found on the same evening, and observations of it continued as long as possible. As it was at no time bright enough to admit of observation in an illuminated field, and there were no means of illuminating the threads of the filar micrometer on a dark field, I was obliged to have recourse to a dark field micrometer. Of the accompanying positions those for August were determined with a square bar-micrometer on the Cooke $4\frac{1}{2}$ -inch Equatorial. On Sept. 1 a ring-micrometer, whose mean radius = $242''\cdot6$, was fitted to the recently mounted Grubb 8-inch Equatorial, and with this instrument observations were continued till Sept. 18, when the square bar-micrometer hitherto employed with the $4\frac{1}{2}$ -inch telescope was adapted to the large instrument. With this micrometer, whose adjustment and errors of form were carefully attended to, the remaining positions were obtained. The comet was at no time a good object for observation, and in consequence of either bright moonlight or haze, such was particularly the case on Sept. 2, 7, 10, 11, 12; Oct. 6, 7, 11, 25 and 29. On Oct. 25 it approached so close to star No. 59 as to be observed with the greatest difficulty. On the whole I think the positions yielded by the square bar-micrometer will be found more satisfactory than those obtained with the ring. Finlay's comet was observed with the large Equatorial from Oct. 8 to Dec. 30.

I may add that the tail of a large comet was visible on the W.S.W. horizon immediately after sunset on the evenings of the 19th, 20th, and 21st inst. at several places in the neighbouring colonies. At Windsor, however, the past few days have been characterised by dense cloud, with rain, so that no opportunity whatever was afforded for getting even a glimpse of the stranger.

1886.	Wind- sor Mean Time.	Comet—Star Δ R.A.	Comets Δ N.P.D.	Comet's App. R.A.	Log. $\frac{p}{P}$	Comet's App. N.P.D.	Log. $\frac{q}{P}$	Red. to App. Place.	Comp. Star.
	h m s	m s	' "	h m s	+	° ' "	+	s	"
Sept. 22	7 20 35	+2 50'96	- 2 54'2	15 31 9'65	8'7141	113 43 24'2	9'5169	+1'38	-0'5 33
22	7 20 35	+1 17'15	+ 6 28'9	15 31 9'58	8'7141	113 43 22'7	9'5169	+1'38	-0'6 34
22	7 20 35	-1 17'10	+ 8 19'5	15 31 9'60	8'7141	113 43 19'5	9'5169	+1'40	-0'7 35
23	7 20 19	+5 3'09	+ 8 29'8	15 36 40'40	8'7143	114 25 13'4	9'5035	+1'39	-0'6 36
23	7 20 19	-2 23'77	+ 3 53'7	15 36 40'64	8'7143	114 25 14'1	9'5035	+1'42	-0'8 37
24	7 24 5	+3 18'67	- 1 26'2	15 42 16'99	8'7192	115 6 40'6	9'4967	+1'42	-0'7 38
24	7 24 5	-4 30'29	+ 7 37'9	15 42 17'21	8'7192	115 6 44'9	9'4967	+1'46	-1'0 39
28	7 7 0	+0 12'53	+ 7 54'6	16 5 31'03	8'6971	117 45 38'4	9'3984	+1'52	-1'2 40
28	7 27 43	+0 18'14	+ 8 23'5	16 5 36'64	8'7245	117 46 7'3	9'4454	+1'52	-1'2 40
28	7 27 43	-5 38'56	+ 0 36'8	16 5 36'68	8'7245	117 46 8'4	9'4454	+1'55	-1'4 41
30	7 18 16	-0 48'34	- 7 19'5	16 17 44'24	8'7121	119 0 51'7	9'3887	+1'57	-1'4 42
30	7 18 16	-2 39'05	- 0 52'2	16 17 44'39	8'7121	119 0 52'0	9'3887	+1'58	-1'6 43
Oct. 1	7 32 2	+1 31'13	+ 0 37'3	16 24 0'75	8'7295	119 37 22'5	9'4055	+1'58	-1'5 44
1	7 32 2	-0 32'46	- 1 0'7	16 24 0'85	8'7295	119 37 20'9	9'4055	+1'59	-1'6 45
5	7 27 28	+0 6'56	+ 2 12'9	8	8'7205		9'3126	+1'68	-2'1 46
5	7 27 28	-4 49'66	- 7 44'4	16 49 42'38	8'7205	121 50 34'1	9'3126	+1'70	-2'3 47
6	7 41 23	+0 12'58	- 5 39'8	5	8'7378		9'3364	+1'70	-2'2 48

1886.	Windsor Mean Time.	Comet—Star Δ R.A.	Comps.	Comet's App. R.A.	Log. $\frac{p}{P}$	Comet's App. N.P.D.	Log. $\frac{q}{P}$	Red. to App. Place.	Comp. Star.
	^{h} ^{m} ^{s}	^{m} ^{s}		^{h} ^{m} ^{s}	$^+$	$^{\circ}$ $'$ $''$	$^+$	^{s}	
Oct. 6	7 42 12	+2 8.75	3	16 56 23.75	8.7388	122 21 13.0	9.3390	+1.69	49
7	8 19 0	-1 59.98	15		8.7776		9.4325	+1.73	50
7	8 19 0	-3 5.90	15		8.7776		9.4325	+1.73	51
11	8 33 5	+1 40.83	13		8.7883		9.4115	+1.79	52
11	8 33 5	-0 50.18	13		8.7883		9.4115	+1.80	53
21	8 47 16	+4 4.68	14	18 40 31.77	8.7859	126 54 51.5	9.3033	+1.93	54
21	8 47 16	-2 18.26	14	18 40 31.71	8.7859	126 54 52.2	9.3033	+1.96	55
23	9 27 0	+3 25.89	12	18 54 30.18	8.8157	127 4 4.5	9.4260	+1.96	56
23	9 27 0	+1 6.78	12	18 54 29.70	8.8157	127 4 6.3	9.4260	+1.97	57
23	10 0 30	-4 5.61	4	18 54 39.03	8.8338	127 4 13.2	9.5278	+1.99	58
25	8 26 59	+9 1.22	6	19 7 45.82	8.7486	127 7 6.4	9.1456	+1.95	58
25	8 26 59	-0 11.82	6	19 7 45.75	8.7486	127 7 7.0	9.1456	+1.99	59
25	8 26 59	-4 0.02	6	19 7 46.17	8.7486	127 7 5.1	9.1456	+2.01	60
25	9 22 13	+0 4.45	8	19 8 2.02	8.8083	127 7 10.3	9.3891	+1.99	59
27	8 27 55	+9 19.18	10	19 21 5.33	8.7419	127 4 27.1	9.1229	+1.97	60
27	8 27 55	+0 15.01	10		8.7419		9.1229	+2.01	61
29	7 43 20	-0 47.22	6	19 33 53.80	8.6503	126 56 36.1	8.7473	+2.02	62

Mean places of the Comparison Stars for 1886.0.

Star.	R.A. h m s	N.P.D. ° ' "	Authorities.
1	13 17 24.38	94 19 38.9	{ Cape Cat. 1850, 2383; Yarnall 5535; Gr. 7 Yr. Cat. 1864, 1569; Glasgow 3366; Gr. 9 Yr. Cat. 1872, 1223.
2	13 18 44	94 19	Approx. Position per Equatorial, Star = 8 mag.
3	13 41 28.14	96 8 5.2	Cape Cat. 1850, 2451; Yarnall 5670; Schj. 4916 and 4917.
4	13 44 39	96 36	Approx. Position per Equatorial, Star = 8½ mag.
5	13 45 45.93	96 36 27.5	Lamont ₃ , 1568; Schj. 4947.
6	13 58 19.29	98 42 34.5	Cape Cat. 1850, 2503; Yarnall 5812.
7	14 0 41.14	98 46 8.5	Gr. Cat. 1850, 883; Cape Cat. 1850, 2511; 2nd Rad. Cat. 1355; Gr. 9 Yr. Cat. 1872, 1285.
8	13 55 35.05	99 23 19.6	Lamont ₃ , 1442.
9	14 5 52	99 25	Approx. Position per Equatorial, Star = 8 mag.
10	14 6 29.01	99 21 48.1	Cape Cat. 1850, 2525; 2nd Rad. Cat. 1365; Gr. 9 Yr. Cat. 1864, 1639; Glasgow 3521.
11	14 12 40.96	101 32 8.6	Schj. 5089.
12	14 21 34.64	102 50 44.6	Cape Cat. 1850, 2570; Radcliffe Obs. 1873, 729, and 1874, 800.
13	14 18 24.38	103 33 47.6	Yarnall 5946.
14	14 19 12.31	103 34 12.2	Yarnall 5955.
15	14 25 2.17	105 6 56.6	Lamont ₃ , 1570. Lalande's position differs considerably from Lamont's.
16	14 30 53	105 1	Approx. Position per Equatorial, Star = 9½ mag.
17	14 39 40.64	104 58 42.2	{ Cape Cat. 1850, 2618; 2nd Rad. Cat. 1418; Yarnall 6071; Gr. 7 Yr. Cat. 1864, 1677; Gr. 9 Yr. Cat. 1872, 1332; Rad. Obs. 1874, 810.
18	14 33 50.33	105 42 24.4	Lalande 26702.

Star.	R.A. h m s	N.P.D. ° ' "	Authorities.
19	14 34 14.67	105 42 52.2	Lalande 26706 and 26707; Lamont, 264.
20	14 38 8.97	106 23 27.5	Arg.-Oeltzen 13891.
21	14 39 39	106 23	Approx. Position per Equatorial, Star = 9 mag.
22	14 59 50	108 41	" " " Star = 8½ "
23	14 59 57	108 38	" " " Star = 8½ "
24	15 1 48.22	109 24 33.4	Arg.-Oeltzen 14280 and 14281.
25	15 5 43.45	109 21 33.9	{ Arg.-Oeltzen 14348; Rad. Cat. 1845, 3329; Cape Cat. 1850, 2695; 2nd Rad. Cat. 1459; Yarnall 6246; Gr. 7 Yr. Cat. 1864, 1710; Gr. 9 Yr. Cat. 1872, 1365; Stone 8261; Rad. Obs. 1874, 832, and 1882, 332; Cape Obs. 1880, 282, and 1881, 438.
26	15 3 17.47	110 4 53.5	Lalande 27567.
27	15 3 43	110 8	Approx. Position per Equatorial, Star = 9 mag.
28	15 19 29.76	110 58 44.4	Arg.-Oeltzen 14544 and 14545; Wash. Mural Cir. Zone 259, 8; Yarnall 6342.
29	15 15 49.37	112 9 24.0	Wash. Mural Cir. Zone 252, 22.
30	15 17 32.55	112 19 0.9	{ Arg.-Oeltzen 14513; Wash. Mural Cir. Zone 252, 24; Yarnall 6328; Washburn-Palermo Cat. 530. 2 secs. have been subtracted from the W. Mural Cir. Zone R.A.
31	15 22 37	112 18	Approx. Position per Equatorial, Star = 9 mag.
32	15 26 33.11	113 4 42.3	{ Arg.-Oeltzen 14643; Wash. Mural Cir. Zone 175, 11. 1 min. has been added to the R.A. of Wash. Mural Cir. Zone.
33	15 28 17.31	113 46 18.9	Wash. Mural Cir. Zone 174, 14; Washburn-Palermo Cat. 540.
34	15 29 51.05	113 36 54.4	{ Wash. Mural Cir. Zone 174, 15; Washburn-Palermo Cat. 542. 1' has been added to Wash. Mural Cir. Zone N.P.D.
35	15 32 25.30	113 35 0.7	Arg.-Oeltzen 14728.

Star.	R.A. h m s	N.P.D. ° ' "	Authorities.
36	15 31 35.92	114 16 44.2	Arg.-Oeltzen 14715; Wash. Merid. Tr. Zone 239, 25. The N.P.D.s are discordant.
37	15 39 2.99	114 21 21.2	{ Arg.-Oeltzen 14840; Cape Cat. 1850, 2831; 2nd Rad. Cat. 1515; Gr. 9 Yr. Cat. 1872, 1406; Stone 8559.
38	15 38 56.90	115 8 7.5	{ Arg.-Oeltzen 14838; Wash. Merid. Tr. Zone 166, 15; Wash. Mural Cir. Zone 165, 62. Arg.-Oeltzen 14974 and 5; Wash. Merid. Tr. Zone 166, 18; Wash. Mural Cir. Zone 165, 66; Washburn-Palermo Cat. 563; Cape Cat. 1850, 2866; Yarnall 6544; 2nd Rad. Cat. 1521; Gr. 7 Yr. Cat. 1864, 1780; Gr. 9 Yr. Cat. 1872, 1416; Armagh 1852; Stone 8628; Rad. Obs. 1882, 353. The Wash. Mural Cir. Z. R.A. and Wash. Merid. Tr. Z. N.P.D. rejected.
39	15 46 46.04	114 59 8.0	{ Arg.-Oeltzen 15351; Cape Cat. 1850, 2957; Wash. Mural Cir. Zone 29, 13; Yarnall 6691; 2nd Rad. Cat. 1552; Stone 8807; Rad. Obs. 1880, 351; Cape Obs. 1881, 468.
40	16 5 16.98	117 37 45.0	{ Arg.-Oeltzen 15482; Cape Cat. 1850, 2992; Wash. Mural Cir. Zone 29, 16; Yarnall 6731; Rad. Obs. 1872, 777; 1874, 900; 1875, 694; 1876, 695; Stone 8858.
41	16 11 13.69	117 45 33.0	{ Arg.-Oeltzen 15609, 10, and 11; Cape Cat. 1850, 3022; Wash. Merid. Tr. Zone 117, 8; Wash. Merid. Cir. Zone 94, 22; Wash. Mural Cir. Zone 263, 16; Yarnall 6784; Rad. Obs. 1872, 786; Stone 8931.
42	16 18 31.01	119 8 12.6	{ Arg.-Oeltzen 15643, 4, and 5; Cape Cat. 1850, 3030; Wash. Merid. Tr. Zone 117, 9; Wash. Merid. Cir. Zone 94, 25; Wash. Mural Cir. Zone 263, 19; Yarnall 6794; 2nd Rad. Cat. 1578; Stone 8941. Wash. Merid. Tr. Z. R.A. rejected. There appears to be a systematic error of 1° in the Right Ascension of this zone.
43	16 20 21.86	119 1 45.8	{ Arg.-Oeltzen 15662; Wash. Merid. Tr. Zone 17, 86; Wash. Merid. Cir. Zone 97, 91; Wash. Mural Cir. Zone 263, 20.
44	16 22 28.04	119 36 46.7	{ Arg.-Oeltzen 15694; Wash. Merid. Tr. Zone 17, 88; Wash. Mural Cir. Zone 263, 24; Yarnall 6809. The authorities are very discordant.
45	16 24 31.72	119 38 23.2	{
46	16 49 34	121 48	Approx. Position per Equatorial, Star = 8 mag.

Star.	R.A. h m s	N.P.D. ° ' "	Authorities.
47	16 54 30.34	121 58 20.8	{ Cape Cat. 1850, 3178; Wash. Merid. Tr. Zone 30, 26; Yarnall 7043; Stone 9253; Rad. Obs. 1880, 370.
48	16 56 9	122 27	Approx. Position per Equatorial, Star = 9 mag.
49	16 54 13.31	122 28 36.6	Wash. Mural Cir. Zone 25, 24.
50	17 5 13	122 49	Approx. Position per Equatorial, Star = 9 mag.
51	17 6 19	122 43	" " " Star = 8½ "
52	17 28 54	124 32	" " " Star = 9 "
53	17 31 25	124 25	" " " Star = 8½ "
54	18 36 25.16	126 49 39.0	Wash. Merid. Tr. Zone 56, 9; Stone 10182.
55	18 42 48.01	126 56 35.0	" " " 56, 11.
56	18 51 2.33	127 15 15.0	{ Cape Cat. 1850, 3716; Wash. Merid. Tr. Zone 56, 13; Wash. Mural Cir. Zone 48, 25; Melb. Cat. 1870, 961; Stone 10309.
57	18 53 20.95	127 13 0.1	Wash. Merid. Tr. Zone 56, 14; Wash. Mural Cir. Zone 48, 26; Yarnall 8056; Stone 10326.
58	18 58 42.65	127 13 32.9	{ Cape Cat. 1850, 3745; Wash. Merid. Tr. Zone 56, 18; Wash. Mural Cir. Zone 48, 28; Yarnall 8108; Melb. Obs. 1880, 302; Stone 10373.
59	19 7 55.58	127 8 38.2	Wash. Merid. Tr. Zone 56, 19; Wash. Mural Cir. Zone 48, 33; Yarnall 8187; Stone 10440.
60	19 11 44.18	127 5 55.7	Wash. Merid. Tr. Zone 56, 20; Wash. Mural Cir. Zone 48, 34; Yarnall 8223; Stone 10459.
61	19 20 49	127 13	Approx. Position per Equatorial, Star = 9 mag.
62	19 34 39.00	126 53 43.9	{ Wash. Merid. Tr. Zone 56, 23; Cordoba Zone 22, 2; Yarnall 8437; Stone 10615. 1' has been subtracted from the first authority for N.P.D.

Windsor, N. S. Wales:
1887, January 24.

Elements of Comet 1886 e (Finlay). By W. H. Finlay, M.A.

The following elements represent my observations of this comet closely. A normal place was deduced from the observations on September 26, 27, 29, and 30, and another from those on December 13, 15, 16 and 17; these were:

Sept. 28.5	$\alpha = 2^{\circ}56'48''38.2$	$\delta = -26^{\circ}10'51''.0$
Dec. 15.5	$\alpha = 336'51''52.2$	$\delta = -10'49''16.0$

From these, by varying the geocentric distances to satisfy the observations on October 21, November 13, and December 27 (on which dates very good places were available for the comparison stars), I obtained the elements

T = Nov. 22.3918, G.M.T.

ω	$3^{\circ}15'54''7.0$	} Ecliptic and Mean Equinox 1886.0
Ω	$52'29''15.2$	
i	$3^{\circ}1'38.6$	
ϕ	$45'51''51.6$	
log α	0.5482066	
μ	$534''.1911$	

The representation of the observations on October 21, November 13, and December 27 is

	C - O	
	$d\lambda$ as β	$d\beta$
Oct. 21	$+0''.7$	$+3''.3$
Nov. 13	$+0.6$	$+5.1$
Dec. 27	-0.8	-2.9

I discontinued my observations when the comet passed to the north of the equator, and was more favourably situated for observation in the northern hemisphere, so that I have no later date than December 27 to compare with the elements. The discordances in latitude are persistent in all the variations of the distances. The heliocentric co-ordinates are

$$\begin{aligned} x &= [9.9996185] r \sin (97^{\circ}32'43''.0 + v) \\ y &= [9.9562980] r \sin (8^{\circ}40'46''.9 + v) \\ z &= [9.6324689] r \sin (2^{\circ}28'49''.5 + v) \end{aligned}$$

Royal Observatory, Cape of Good Hope:
1887, February.

Comet 1887 *a*. By W. H. Finlay, M.A.

The tail of this remarkable comet was first seen, so far as I am aware, by a farmer and a fisherman at Blauwberg (near Cape Town), on Tuesday night, January 18; the next evening it was seen at Grahamstown, Fraserburg, &c.; our first view of it at the observatory was on January 22. It presented the appearance of a pale narrow ribbon of light, quite straight, and of nearly uniform brightness throughout its length. There was no head or condensation of any kind visible near the end, the light simply fading away to nothing. The comet was lost in the 6-inch Equatorial long before the end of the light, as visible to the naked eye, was reached. The following rough observations were made by tracing the tail down as far as possible with the finder of the 6-inch and reading the circles of the Equatorial. On January 24 and 27 the circles were also read for the centre of a very slightly more condensed part of the tail, extending over about half a degree. A star near the same position was also observed in a similar manner each evening. The resulting places are:

Date.	Cape M.T.		App. R.A.			App. Dec.	
	h	m	h	m	s	°	'
Jan. 22	8	50	21	30	0	-45	47½
23	8	44	21	47	54	-46	32½
24	8	50	22	8	15	-47	35
24*	8	55	22	9	20	-47	54½
25	9	4	22	30	44	-48	39
27	8	58	23	16	10	-49	38
27*	9	35	23	17	11	-49	50
28	9	38	23	37	12	-49	22

The observations marked * are of the centre of the slightly brighter part of the tail. Moonlight put a stop to any further observations. The tail was sketched each night on a map copied from Gould's *Uranometria*; it was quite straight at first, but on the 27th, 28th, and 29th a slight curvature was perceptible.

The physical appearance of the comet, its long straight tail of no greater brilliancy than the smaller Magellanic cloud, and the absence of head at once recalled to mind the comet of February 1880, and as soon as I had secured a place on January 22 I tried whether the observed place could result from the elements of that comet. I found that it could very nearly with a true anomaly of $168^{\circ}7$ and perihelion passage

January 11^o, and the comet's subsequent motion is very similar to what would result from these elements.



1887, January 22, 8^h 50^m C.M.T.

" 23, 9^h 5^m "
 " 24, 9^h 15^m "
 " 25, 9^h 30^m "

January 27, 9^h 15^m C.M.T.

" 28, 9^h 35^m "
 " 29, 9^h 15^m "

From the observations on January 22, 25, and 28 I find the following orbit :

$T = \text{Jan. } 11^{\text{d}}.244, \text{ G.M.T.}$

$\pi \quad 89 \quad 41$
 $\varpi \quad 359 \quad 41$
 $l \quad 141 \quad 16$
 $q \quad 0.0146$

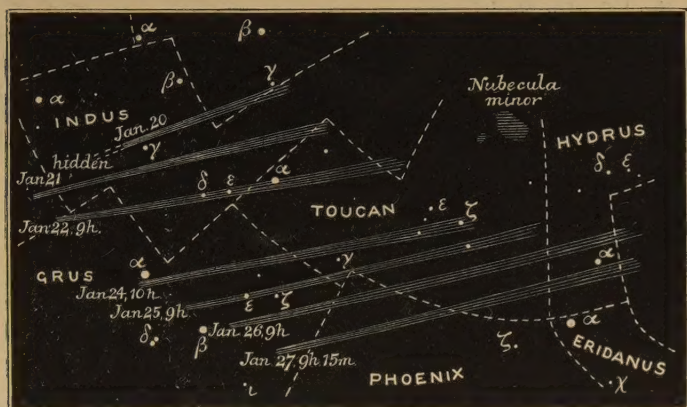
This orbit satisfies the middle place in longitude, but leaves a discordance of 4' in latitude. These elements, though of course rough, prove conclusively that the comet belongs to the family of 'Sun-grazers,' of which 1843 I., 1880 I., and 1882 II. are members.

Royal Observatory, Cape of Good Hope :
 1887, February 14.

Observations of Comet 1887 a, made at the Observatory, Adelaide.
By Charles Todd.

(Extracts from letters to the Astronomer Royal.)

A few words about the new comet which has suddenly burst into view in the S.W. I sent you a telegram through Mr. Hesse on Friday, the 21st, which I hope you received, and I now enclose a rough sketch-map showing its positions on Jan. 20th to Jan. 27th. It was first seen on the 19th, but at the Observatory, by myself, not till the 20th, shortly before 9 P.M. The head of the comet was then hid by the mists of the horizon, but the tail passed nearly midway between α *Gruis* and α *Pavonis*, or over γ *Indi*, extending to and a little to the right of γ *Pavonis*.



On the 21st it was better seen and the head of the comet was close to a small star, or roughly (at 9 P.M.) R.A. $21^{\text{h}} 5^{\text{m}}$, Decl. $42^{\circ} 34'$. The tail of the comet, as measured round the horizon, had sensibly moved to the north or towards α *Gruis*; it passed a little to the right of γ *Indi*, and to the left of α and δ *Toucani*, and could be traced a little higher in the direction of β *Hydri*. On the 22nd, at $8^{\text{h}} 30^{\text{m}}$, the position of the head of the comet was roughly R.A. $21^{\text{h}} 20^{\text{m}} 30^{\text{s}}$, Decl. $44^{\circ} 17'$. The comet has a long narrow tail of about 30° , but no well-defined nucleus, resembling, in fact, very closely in appearance the comet of Feb. 1880, which during its short season of visibility, you will remember, occupied the same part of the sky, and came into view equally suddenly. I will follow it up on every clear night and advise you again. In the meantime this will be useful in comparing with observations made elsewhere, and may, perhaps, be of some use in getting a rough orbit.

The following are the only positions of the comet I was able to secure before the Moon shut it out. Although a conspicuous object, about as bright at first as the Magellan clouds, it became rapidly fainter, and at no time was there any defined nucleus* or condensation about the head. It was simply a narrow nebulous streak of about 30° long, and although with the naked eye, or with an ordinary field-glass, the position of the head of the comet could be fairly determined, yet seen through the Equatorial the head was so diffused and died away so gradually that it was quite impossible to say where it was. The positions I give are consequently mere rough approximations obtained by the Equatorial, checked by the naked eye.

On the 24th, at 9 P.M., the head of the comet was partly over and a little to the right of *a Gruis*, which star appeared as a bright stellar nucleus.

On the 20th the head was in the midst of the group τ^1, τ^2, τ^3 *Gruis*, the tail passing between $\eta 2$ and 58, and over *Toucani* and to the left of *a Hydri*.

On the 27th the head of the comet, a very diffused nebulous mass, appeared to be cut off by a wide break or rift, from the tail, and higher up there was another narrow break in the continuity of the tail. Both of these breaks might be attributable to small clouds, but we could see faint stars through the breach which seemed to forbid such a supposition :

	h	R.A.		Dec.
		h	m	
Jan. 21	9	21	5	-42 34
22	9	21	21	44 17
25	9 $\frac{1}{2}$	22	22	48 18
26	9.40	22	47	49 14
27	9.15	23	9	49 39 guess work

I shall be curious to know how these agree with, perhaps better, positions obtained elsewhere. I send them hoping they may at least be useful in identifying the comet which I am disposed to regard as a fragment of some previous visitor.

The Moon and Aldebaran, 1887, March 2, 5^h 45^m to 6^h 0^m G.M.T., as seen at the Radcliffe Observatory, Oxford. By E. J. Stone, M.A., F.R.S.

The occultation of *Aldebaran* by the Moon predicted to take place on 1887, March 2, was carefully looked for at the Radcliffe Observatory, but though an extraordinarily near appulse took place there was no occultation at Oxford. The overlapping of

* A nucleus was observed by Mr. Eddie of Grahamstown, on Jan. 24, 25, and 26.—Ed.

the image of the Moon's limb and the diffraction image of the star appears to render a record of these observations desirable.

The following notes were made by the observers:

Mr. Wickham, with heliometer, $7\frac{1}{2}$ -inch object-glass, and power = 200:

'The star did not disappear, but skirted the north limb. The exterior flames of the star's image on the side next the Moon were, at times, lost in the Moon's disc, although there was a strong contrast between the quality of the Moon's light and that of the star. The Moon's colour was a dead creamy white, the star's a brilliant light orange. The close contact of the star's disc to that of the Moon lasted without apparent change for nearly three minutes. The nearest appulse took place close to the division of the illuminated and defective limbs. There was very strong twilight.'

Mr. Robinson, with Marlborough telescope, $3\frac{1}{4}$ -inch object-glass, and power = 80:

'The star was not occulted, but it apparently grazed the Moon's N. limb: three or four minutes previous to this, or about $5^h 47^m$ G.M.T., the Moon's periphery when produced beyond the N. cusp appeared to bisect the star. The dark limb of the Moon was not visible owing to twilight. The colour of the star greatly contrasted with that of the Moon, being orange-red throughout.'

Mr. Bellamy, with 10-foot telescope, 7-inch object-glass, power 125:

'The star was not occulted, but the outer rings of the image were, at times, in contact with and a small portion lost sight of in the illumination of the Moon's limb. The time when I estimated that the star was nearest the bright limb of the Moon was about $5^h 51\frac{1}{2}^m$ G.M.T.'

Note on the Near Approach of Moon to a Tauri.

By Thomas Gwyn Elger.

Though this object was not actually occulted here on March 2, the exceedingly near approach of the Moon and star was watched with much interest, as it seemed probable, when the time drew near, that it might suffer a momentary eclipse by some of the elevations on the Moon's northern limb. Between $5^h 49^m$ and $5^h 54^m$ it appeared to be almost in contact with one of these, without, however, its light being visibly affected. The position of my observatory, according to the 6-inch scale map of the Ordnance Survey, is N. Lat. $52^\circ 7' 10''$, W. Long. 2 mins. 0.4 sec. A power of 150 was used on a $8\frac{1}{2}$ -inch Calver Reflector.

Kempston, Beds:

1887, March 5.

Second Occultation of Aldebaran, 1887. By the
Rev. S. J. Johnson.

In marked contrast to the occultation period of 1866-9, when only one emersion of *Aldebaran* was visible from my place of observation and no immersion, the second passage of the Moon over the star visible here this year, on March 2, was seen under a sky even more favourable than the first. Parallax slightly lengthened the duration as compared with Greenwich. Immersion $5^h 40^m 53^s$, quite as instantaneous as last January. Sun had reached the horizon but had not descended. At $6^h 0^m 56^s$ *Aldebaran* clear of the Moon's limb, but visible on the bright edge five seconds before this. The illusion conveyed to the eye at emersion, probably from the smallness of the instrument and power, 50 on $3\frac{1}{4}$ inches, was that the star was shining through the bright limb. The only time I have had this impression before was during the totality of the eclipse of August 23, 1877, when three or four small stars appeared within the reddened disk for 8 or 10 seconds before actually disappearing. The same impression appears to have been conveyed to the eyes of observers at Greenwich and elsewhere on the occasion of the occultation of October 15, 1829.

Melplash Vicarage, Dorset :
March 9.

The Occultation of Aldebaran. By C. Leeson Prince.

I observed the occultation of *Aldebaran* this evening under very favourable circumstances; the Moon being so near the meridian, the Sun had just set, and the atmosphere was very calm, clear, and diaphanous; nevertheless, the non-illuminated portion of the Moon was not visible.

The disappearance occurred at $4^h 25^m 57^s$, and the reappearance at $4^h 48^m 37^s$, local sidereal time. The star was occulted, therefore, exactly $22^m 40^s$.

The disappearance was absolutely instantaneous, but the reappearance not quite so, a very minute point of light being visible for nearly a second before the star completely emerged; this may perhaps have been occasioned by a slight elevation on the lunar surface at the exact point of reappearance. Telescope 6.8 inches aperture, focal length, 12 feet; mag. power, 144.

The Observatory, Crowborough, Sussex :
1887, March 2.